Note sur la loi des grands nombres[*]

M. Poisson

Comptes Rendus hebdomadaires II (1835), pp. 377-382

"In the preamble that I have read some months ago to the Academy,[1] from the work on the *Probabilité des Jugements criminels* of which I am actually occupied, I have considered the *law of large numbers* as a fact that we observe in things of all natures. The varied examples that I have cited of it would not be able to leave any doubt on its generality and its exactitude; but seeing its importance, it was necessary to discover the principal of it, and to demonstrate it directly. I am in fact arrived to it, in the same way that one will see it in this work, of which the impression shortly is going to begin; and the principal object of this note is to announce this result which seems to me ought to interest the geometers.

"One must not confound this general law with the beautiful theorem due to Jacob Bernoulli, who meditated on the demonstration of it, as one knows, during twenty years. According to this theorem, the events arrive very nearly, in a long series of trials, proportionally to their respective probabilities; but one must not lose from view that he supposes that these chances remain constant; now, on the contrary, the chances of physical phenomena and of moral things, vary nearly always continuously without any regularity, and often in a great extent; however, a constant observation shows us that for each nature of events, the ratio of the number of times that they arrive to the total number of trials is sensibly invariable, when these numbers are very great, so that this ratio appears to converge in measure as these numbers increase yet further, toward a special magnitude that it would attain if the trials were able to be prolonged to infinity. It is also this which the theory demonstrates rigorously, without making any hypothesis on the law of variation of the chances, and independently of the nature of things, moral or physical. When one considers the irregularity of chances and their more or less great variations during a long series of observations, the constancy of the observed ratios among the great numbers for each sort of events, a surprising thing is able to appear that one is tempted to attribute to some general cause and unceasingly active; but the theorem shows that this permanence is the natural state of things, which are maintained by themselves without the help of any strange cause, and which, on the contrary, would have need in order to change, from the intervention of a parallel cause. One is able to compare this state to the repose of the body which subsists by virtue of inertia alone of matter, as long as no strange cause comes to trouble it.

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[1]*Comptes rendus hebdomadaires*, I, page 473.

"In order to give, by a simple example, a precise idea of the difference which exists between the law of large numbers and the theorem of Jacob Bernoulli, I suppose that one projects 2000 times in sequence one same coin of 5 francs, and that one of the faces arrives 1100 times and the other 900 times; the common chance, *a priori*, of the arrival of the one or the other of these two faces, is here invariable, since it depends on the physical constitution of the coin which does not change during the trials; it suffices therefore from the theorem cited in order to conclude that this chance is nearly $\frac{11}{20}$ for one of the faces and $\frac{9}{20}$ for the other, and in order to conclude that if one repeats these trials again a very large number of times with the same coin, the first face will arrive very nearly $\frac{11}{20}$ of this number of times, and the second $\frac{9}{20}$. But if one projects successively 2000 different coins of 5 francs, the chance of each of the two faces will be without doubt not the same for all these faces, and the theorem of Jacob Bernoulli will no longer be able to be applied; nevertheless, if one of these faces arrive, in order to fix the ideas, 1200 times and the other 800, one will conclude from the law of large numbers that in a new sequence of a very great number of trials, made with some 5 franc coins of the same fabrication as the first ones and with the same effigy, these two faces will arrive still some number of times which will be among them quite nearly as 12 to 8.

"This material example is a picture of that which happens in moral things, considered independently of the nature of their causes, and only as for their effects. In criminal judgments, for example, the condemnation and the acquittal of the accused have some chances which vary from one process to another, likewise as the chances of the two faces of the coins of 5 francs, change from one coin to another. Now, that does not prevent that in some very great numbers of process, the ratio between the number of acquittals and that of the condemnations are very nearly invariable, as well as the ratio between the number of arrivals of the two faces of different coins. Thus, during six consecutive years that the legislation on the jury has not changed in France, the number of acquittals has been annually 0.39, average term, of the number of the accused: one time alone it is elevated to 0.40, and one time alone it is lowered to 0.38. It has changed next with legislation. In Belgium, since the restoration of the jury toward the middle of 1831, this ratio has been 0.41, 0.40, 0.39, for the three years 1832, 1833, 1834. Before, it was moreover less than half, and was elevated only to around 0.18. The criminal tribunals which judged without intervention of juries, were composed of five judges, and were able to condemn by the simple majority of three against two. Out of ten accused, they condemned with eight, instead of six as the jury condemns today. One is able to consult on this point the *Comptes généraux de l'administration de la justice criminelle en Belgique*, recently published by the government of this realm.

After the communication of M. Poisson that one just read, a discussion is raised within the Academy, on the subject of application of the calculus of probabilities to some questions of the moral world. Messers Poinsot, Dupin, Navier have successively taken the podium. Here is the succinct résumé of the opinions enunciated by these three academicians.

> "The calculation of probabilities in moral things, such as the judgments of tribunals, or the votes of assemblies, seems to M. Poinsot a false application of mathematical science; he thinks that one is not able to draw

any consequence that may be able to serve to perfect some decisions of men. According to Mr. Laplace himself the *theory of probabilities holds to some considerations so delicate, that it is not surprising with the same givens that two persons find some different results especially in very complicated questions*; whence Mr. Poinsot would be able to conclude that the theory of probabilities is so delicate that it is very probable that the geometers are deceived often in this analysis; so that after having calculated the probability of error in a certain thing, it would be necessary to calculate the probability of error in his calculation. This idea alone of a calculation applicable to some things where imperfect knowledge, the ignorance and passions of men are mixed, are able to make a dangerous illusion for some minds, and it was precisely this consideration which Mr. Poinsot had determined to take a moment the podium on a question so little geometric."

Observations of M. Charles Dupin — Our colleague Mr. Poisson seems to assure us that by reuniting a very great number of judgments by jurors, the disproportions that one is able to note out of some small numbers among the acquittals and condemnations, diminish and erase themselves, so to speak, independently of all social circumstances.

"In this regard I make a first observation: it is relative to the enormous differences between the acquittals and the condemnations by the actual jurors and by the jurors of the revolutionary tribunal.

"Yes, sirs, with difficulty you would find for average some acquittals of this tribunal, 5 against 100; while our actual tribunals present, according to the same numbers reported by our scholarly colleague, 39, 40, and 41 acquittals against 61, 60 and 59 condemnations.

"But that which is more remarkable and more deplorable, it is that the acquittals, which, in the principal of the revolutional tribunal, were in a proportion much less small, diminish in measure as the number of judgments increase, although the chances of composition of the jury remain the same.

"Mr. Poisson responds that he has considered this case; which he has found for the proportion of the acquittals to the condemnations out of a very great number of judgments two roots of an equation: the first which agrees to the times and to the ordinary cases, the second which agrees to the times and to the extraordinary cases, such as the times of the terror and the case of the revolutional tribunal.

"I will make a remark to the Academy which enters the cases of the extreme terror and those of a perfect security, the social state, in its phases, offers us successively all the intermediate degrees of insecurity and of intimidation.

"Here is therefore a vast sequence of causes which tend to change very notabley the proportion of acquittals to condemnations, whatever be the number of the accused of each period.

"Yet another observation. It is of the shy and susceptible administrations, which themselves make a system to multiply the losses in accusation. Ah well! every time that there is not terror exercised by the power, the jury responds in contrary sense; not only it clears the slight guilts, but by analogy it repels to condemn the grave offences. Thus, in this case again, the more the number of judgments increase, the more the proportion of acquittals to condemnations become considerable. The disproportion is

even able to go so far as the administration recurs to the laws of exception in order to retire to the jury of the causes of a certain nature, and in order to change the proportions of the vote and its conditions of secrecy or of publicity.

"Now, I repeat it, in the midst of chances so variable and of circumstances so powerful, how can one hope to arrive to some *limit proportions*, which one will be able to regard with some benefit such as from the *mean terms* to take for base of some reasonings? finally when from large sequences of particular cases they will be able to deviate very considerably from this limit, to what these mean terms will serve, and what applicable consequences one can hope to deduce from it?

"Mr. Poisson has made a remark to us that, in the middle of France, the proportion of the acquittals to condemnations is certainly less than in the north.

"Consequently, if it operated distinctly out of the two parts of the realm, there would arrive for some very great numbers some limit proportions which would differ very sensibly, although the composition and the mode to operate of the jury are identical in the two parts of France.

"And if he takes France entire, he goes to find a third limit proportion which will represent neither the north nor the south, but an ideal mean term: surely, such is not the idea that one is able to form of a final term toward which the solutions of all the juries of a country gravitate."

M. Navier demands to present some remarks relative to the distinction which one has seemed to establish between the natural facts, of which one would regard the ones as being subject to some invariable laws, and the others as being entirely fortuitous and accidental, and hence as they are not able to hold with some investigations based on some rigorous methods. "Mr. Navier thinks that the facts of each kind out of which our observations are able to carry, and even the political or judicial facts in which the human passions and interests intevene, depend equally on determined and subsisting laws, founded on the nature of Man. This principal being admitted, one will conclude necessarily from it that attentive and regular observation of the facts are able to shed some lights on the events to come, by putting into evidence the effects of the laws of which there is question, and to lead to establish some results to which one will be able to accord a certain degree of confidence, of which the calculus of probabilities has especially for object to give the measure. The greater part of the objections that some persons present against this calculus, hold besides to that which they suppose that one claims by its mean to be in a state to assign that such or such event will take place; while the results to which the calculus of the probabilities lead, never consist but in the evaluation of the diverse probabilities which belong respectively to many previous events, and of which the possibility is admitted.