INAUGURAL DISSERTATION
MATHEMATICAL-JURIDICAL

ON

THE USE OF THE ART
OF CONJECTURING
IN LAW,

Which

BY AID OF DIVINE GRACE
By the Authority and by the Command

Of the most Magnificent & most Distinguished Order of
Counselors of Law in accordance with the Fatherland's
University

for the

DOCTORATE DEGREE

In both Roman and Canon law legitimately acquired

On the day 14 June A.C. MDCCIX

L.H.Q.S.

Publicly he will defend

M. NICOLAUS BERNOULLI,

Basel

To the most distinguished & most knowledgeable man
Master J. Jac. Battier,

Doctor of Canon and Roman Law,
at the Imperial Institute & Professor of Public Law,
most meritorious, of the Academy here the Magnificent Director,
my own Patron and Teacher to be revered continuously:

And also

TO THE MOST FAMOUS,
MOST EXCELLENT, MOST CELEBRATED MAN
MASTER JOH. BERNOULLI,

Philosopher & Doctor of Medicine, Professor of Mathematics at Basel
Both The Royal Society of Sciences of France & The Prussian Society
Most distinguished Member,
my own most respected Uncle

This specimen in Mathematics-Law
In indication of a grateful spirit, and of his higher
recommendation,

these things for which attention is proper
he presents and dedicates

NICOLAUS BERNOULLI
Nicolaus, son.

PREFACE

With the usual Examinations having been endured just previously for some months, to which the Candidates of Law at Basel are subjected, I have hoped that I would make do by no means ineptly, if in the place of the fourth evidence, which still remains to be determined, by public debate, I would discuss some theme out of mathematics, produced by that divine knowledge, the study of which I have joined so far with the study of law with GOD favoring,and how from the first years I have continued with conspicuous love, with my most celebrated Uncles Jakob & Johan Bernoulli displaying a light for me in this knowledge, the first of whom, now indeed enrolled in the heavenly chorus of the blessed, to his own whom he has bequeathed (unedited thus far but shortly, as we hope, brought into light) the Treatise on the Art of Conjecture, he has made available to me the opportunity, of choosing this material, concerning the Use namely Of the Art of Conjecture in Law, which also there I undertake with pleasure, because I see, that many of the most useful investigations, particularly about absent men to be considered dead,likewise life annuities &c. occurring nearly daily in the Court of Justice,by this art are able to be decided.
Thus by reason of the exposition having been explained with a few words, I approach the matter itself without further delay.

It would be consequently.

J. N. D. N. J. C.

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DISSERTATION
ON
THE USE OF THE ART OF CONJECTURE IN LAW

Chapter 1

THE ART OF CONJECTURE IN GENERAL

Following Cicero's advice at *de Offic.* 1.1[1] every discipline which takes its beginning from another one, so that it may be rationally understood, ought to proceed from a statement of what it might be, from which <vantage point> it can be contested; indeed in the first place saying what might be the Art of Conjecture favors us. Moreover it is a very great pleasure to define this <Art> in accordance with the very words of my Uncle,[2] my master, p(rofessor) m(athematics) in the *Tractate on the Art of Conjecture Part. IV Chapter II* as far as it would be the

> *Ars metiendi, quam fieri potest exactissime, probabilitates rerum eo fine, ut in judiciis & actionibus nostris semper eligere vel sequi possimus id, quod melius, satius, tutius aut consultius fuerit deprehensum.*[3]

The goal of this Art, as the definition makes clear, is warranted by the random uncertain and doubtful matters, in which although all persuasive certainty is not possible to be held, we are nonetheless able to delimit through conjecture, how great the probability would be, such as this or that might be or could happen, or what probably might be about to exist, or which outcome would be 5.

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[1] "My dear son Marcus, you have now been studying a full year under Cratippus, and that too in Athens, and you should be fully equipped with the practical precepts and the principles of philosophy; so much at least one might expect from the preeminence not only of your teacher but also of the city; the former is able to enrich you with learning, the latter to supply you with models. Nevertheless, just as I for my own improvement have always combined Greek and Latin studies – and I have done this not only in the study of philosophy but also in the practice of oratory – so I recommend that you should do the same, so that you may have equal command of both languages. And it is in this very direction that I have, if I mistake not, rendered a great service to our countrymen, so that not only those who are unacquainted with Greek literature but even the cultured consider that they have gained much both in oratorical power and in mental training. Cicero, *De Officiis*. Trans. Walter Miller. *Loeb Classical Library* Vol. XXI, 1921.

[2] James I (Jakob) 1654-1705.

[3] The art of measuring, as precisely as possible, probabilities of things, with the goal that we would be able always to choose or follow in our judgments and actions that course, which will have been determined to be better, more satisfactory, safer or more advantageous.

more probable than another, or how much this or that might deviate from an integral certitude; I very much intend *integral*: for probability is the gradation of certainty and it differs as much from this <i. e. certainty> as the part from the whole. If certainty is indisputably integral & absolute, which we designate by the letter *a* or number 1, e.g. it might be supposed to consist of 5 probabilities, or rather "parts", of which three would argue strongly for the <actual> existence or the future occurrence of an event, the remaining <two parts would argue forcefully> against <their occurrence>; that outcome is said to have a $3/5a$ or $3/5$ probability. This therefore is called more *probable* than the other because it has the greater claim to certainty; although this would be only referred to normally as *probable* in a positive degree, whose probability exceeds significantly 50% certainty. I stress *significantly*; for what approaches a 50% certainty is called *uncertain* or *doubtful*. Thus what has a $1/5$ certainty is more probable than what has $1/10$, although neither may be probable in a positive degree as my uncle taught in the aforementioned Tractate *Part. IV Chapter I.*

This <essay is relevant to> the goal of the Art of Conjecture because we have said that there are uncertain or doubtful matters: From these matters moreover, which are certain & whose truth can be apprehended easily, this is not done; when following the first <matter> according to the Rules, which Uncle wrote in Chapter II, there ought not to be a place for conjecture in <assessing> matters in which one may comprehend overwhelming certainty. Thus if a thief upon interrogation will have responded that he sold stolen property to Sempronius,[4] a judge would try the case ineptly, who from a look or tone of voice, or from the quality of the merchandise stolen by the thief, or from other circumstances of the thief would want to conjecture concerning the probability of the assertion, when Sempronius is present, from whom it will be allowed that everything can be discovered certainly & easily.

The foundation of this entire Art, upon which we ought to rely perpetually in assessing probability, in this general Rule consists, which Huygens[5] demonstrates in his elegant *Pamphlet de Ratiociniis in aleæ ludo*,[6] *Propositions* 1, 2 & 3 and my Uncle in his *Notes to these same Propositions.*

> *Multiplicetur id quod singulis casibus evenit per numerum casuum, quibus unumquodque evenire deprehenditur, summaque productorum dividatur per summam omnium casuum, quotiens ostendit quid probabiliter eventurum sit, sive denotabit valorem expectationis seu gradum probabilitatis quæsitæ.*[7]

The Rule is the same as that in which commonly the *mean arithmetical proportional* is sought among many more given quantities, and indeed with the

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[4]Sempronius is a name commonly used in the legal literature for a party in a legal action.

[5]Christiaan Huygens 1629-1695.

[6]This treatise published in 1657.

[7]That would be multiplied which occurs in individual cases by the number of cases in which one or another <possibility> is considered to occur, and the sum of the products would be divided by the sum of all the cases, the quotient indicates what probably will occur, either it will indicate the value of the expectation or the degree of the sought-out probability.

rule of *alligation*,[8] on which matter it is a pleasure to offer the notes of Uncle my master, which he has in his *Notes to Proposition 3 of the Pamphlet of Huygens.* 6.

> *Perspicuum est ex calculi hujus consideratione, magnam illi inter-cedere affinitatem cum Regula Arith. Alligationis dicta, qua res diversi pretii in data quantitate miscentur, & quæritur pretium rei mixtæ; aut potius calculum utrinque plane eundem esse. Sicut enim summa productorum ex quantitatibus singulorum miscibilium in sua respective pretia, divisa per aggregatum omnium miscibilium, exhibit pretium quæsitum, quod semper medium est inter pretia extremorum: ita summa productorum ex numeris casuum in id quod quovis casu acquiritur, divisa per numerum omnium casuum, ostendit valorem expectationis, qui proinde semper intermedius erit inter maximum & minimum quod acquiri potest. Unde si iidem numeri assumantur, ibi pro quantitate miscibilium, eorumque pretiis: hic pro casibus,& eo quod quovis casu obtinetur; idem quoque numerus denotabit ibi pretium rei mixtæ,& hic expectationem. Ex. gr. si 3 canthari vini pretii 13 misceantur cum 2 cantharis pretii 8; multiplicatis 3 per 13 & 2 per 8, exurgit pretium omnium cantharorum 55, quo diviso per 5 numerum cantharorum, habetur 11 pretium unius cantari mixti: quanta quoque juxta regulam expectatio cujuspiam æstimanda est, qui 3 habuerit casus ad 13, & 2 ad 8.*[9]

But yet that individual and excellent agreement deserves to be noted, which this Rule has with it, which is said <to be> of the greatest importance in regard to finding the center of gravity. For just as the sum of the moments i.e. the sum of the products of the weights in their respective distances from some given fixed point, <that sum> divided by the sum of their weights, shows the distance of the center of gravity, i.e. of that point from which the weights are suspended in equilibrium: Thus even this midpoint, which is obtained by the present Rule, is, as thus I would say, the *center of gravity* of all probabilities, which thus places

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[8]The so-called Rule of Mixtures.

[9]It is evident from a consideration of his calculation that to him there is a great affinity with the Arithmetic Rule of *Alligation*, by which products of a different price are combined in the given quantity, & a price for the mixed product is sought: or rather the calculation done both ways is essentially the same. For just as the sum of the products from the quantities of individual constituent commodities in their own prices respectively, when <that sum> is divided by the aggregate of all the constituent commodities, it exhibits the sought after price because it is always the mean among the prices of the extremes: thus the sum of the products from the number of cases in that which is acquired by any case, when it <the sum> is divided by the number of all the cases, exhibits the value of the expectation, which will be always likewise intermedial between the minimum and maximum which is able to be acquired. Whence if these same numbers may be assumed, <they are constant> there in regard to the quantity of the constituent commodities and also in their prices; they are constant here in regard to cases and <in regard to> that which is obtained by random chance; so too the same number will represent the price of the mixed product & this <represents> the expectation. For example, if 3 canthari of wine priced at 13 are mixed with 2 priced at 8; by multiplying 3 by 13 & 2 by 8, the gross price of the canthari is 55, dividing by 5 canthari, the price of one cantharus of mixed wine is 11: Also how great the expectation of anything must be estimated close to the rule, in order that 3 will have had chances up to 13 & 2 up to 8.

these <things> in equilibrium, so that neither of these probabilities, which fall away on both sides from this mean, takes the greater weight for itself in turn.

Our legal experts intending to preserve such an equilibrium in doubtful and vexing <cases> ought to follow the *mean* as seems apparent from *l. 3. ff. si pars her. pet.* there:

Prudentissime juris auctores medietatem quandam secuti sunt;[10]

and moreover in the same *d. l. 3* they shall have pursued the mean precisely, as we shall see below. This also pertains to this degree to the proverbial expression:

Semper in obscuris, quod minimum est, sequimur.[11]

l. 9 ff. de R. J. c. 30. de R. J. in 6to, a similar thing is said in *l. 115 ff. c. 45 in* 6to that one ought to determine in problematic cases what is most likely to be true or what has the greater claim <to be true>. For this law of ours shows where there is even the least danger of deviating from the truth: in the *middle* evidently outside of which all other probabilities yield more towards the extremes, i.e. they incline more to those things which *most rarely* happen.

7.   Finally this must moreover be noted which my Uncle cautions in *Scholium to Proposition I of Huygens' de ratiociniis in ale ae ludo*; the word *expectation*; in some manner we have said that that mean which is obtained by dividing the sum of the products from the cases in it which are attained by some kind of chance, by the number of all cases, cannot be taken here to mean in its ordinary sense, in which we are said to expect or hope generally that which is the best of all, granted that for us something worse can happen, but to what extent our hope of obtaining the best has been tempered and diminished by fear of a worse outcome: so that something is always represented by the valuation of that which is intermediate between the best we can hope and the worst we fear. Thus he who has 3 chances to gain 13 & 2 to gain 8 cannot be said to expect 13, but 11 which is mean between 13 & 8.

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[10]Most prudently judges have followed a certain mean. See Chapter VIII.

[11]We always follow in problematic <cases> what is the minimum.

Chapter 2

CONCERNING HOW THE PROBABILITY OF HUMAN LIFE SPAN IS COMPUTED, OR RATHER OF A MAN OF WHATEVER AGE

In the prior chapter we have considered the art of conjecture in general, there follows how we make evident the use of it through some particular examples. First what from this Art I have deduced, & in the following it is possible to have utility not deserving of scorn, it is the estimation of the longevity of human life; for although the end of our life be most uncertain, & the hour of death is known to no one except GOD the highest and best the ultimate giver of our life, who is able to take from us this his own gift, at whatever time he himself shall have pleased, he is able to take away; nothing remains for us other, than that through conjecture we would determine, how many years up to this time of the lifetime of anyone at all a man would probably gain, or how much be the probability that he would exceed some given year or not &c.

I see however that, there are many who will oppose in the case of the beginning itself immediately & they will say, not only that it is impossible, that all this can be estimated according to the Art of Conjecture, for there is required the exact enumeration of all chances in which anything is able to happen, but there is no one of mortals, who could ever define the number, e.g. of diseases, as just so many chances, which invade the countless parts of the human body at any age, and they are vigorous to inflict death upon us, and he would know how much easier this than that, plague than dropsy, dropsy than fever kills a man, so that thence concerning the future state of life and death a conjecture is able to be formed, since all this depends upon causes entirely hidden & on knowledge removed from our perception; indeed in another way the matter reveals itself to be the subject in divination & games, which fate alone governs, for since in these the expectation is able to be determined precisely and scientifically, because we accurately & clearly perceive the number of chances, according to which infallibly there ought to follow profit or loss, & since these chances manage themselves indifferently, & they would be equally likely to happen, at least if one should be more probable than the other, we are able to define scientifically, how much

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more probable it would be.[1]

To this we shall respond, that for us another way here of investigating the number of chances is sufficient, which if not *a priori* or from reason, at least *a posteriori* or from an event observed many times in similar examples it will be allowed to bring to light (for I am less able to stray from true proportion, if more frequently, than more rarely I should observe <something>); since it ought to be presumed, that in so many chances each one in the future can happen & not happen, how often previously in a similar condition of the events it will have been discerned to have happened & not to have happened.

For if e.g. after having once conducted an experiment on three hundred men, of the same age and physical constitution, of which Titius is now, you will have observed that two hundred of them died before exactly ten years have passed, <while> the remaining prolonged life beyond, safely you will have collected enough, that it is twice as likely, that by them & by Titius that the debt to nature must be paid within ten years, rather than that it is possible for them to live beyond that limit.

Hence it is manifest what must be realized about the excellence of the Art of Conjecture, for how much less those things, which are fortuitous and also uncertain, seem to be able to be comprehended within the limits of reason, so much more admirable the Art must be valued, to which lesser things are also subject, as Huygens says to Schootens[2] *in the preface of his pamphlet De ratiociniis in aleæ ludo.*

The truth of his assertion reveals itself very clearly thus far, if now truthfully we shall have revealed the method of raising the probability of human life by means of a calculation drawn from observations made concerning the Bills of mortality, of which kind are accustomed to be distributed in Paris & London monthly or weekly. Uncle, my distinguished master, p(rofessor) m(athematics) refers in *Dissertatione de Conversione & Oppositione Enunciationum annex 31.*[3] from *Ephemeris Erud. Gall. Anni. 1666 Num. 31.*[4] that it has been observed from the collection of many such bills, that out of one hundred infants born at the same time there remain surviving 64 after six years have elapsed, after XVI years have elapsed 40, after XXVI years 25. XXXVI years 16. XLVI years 10. LVI years 6. LXVI 3. LXXVI years 1. LXXXVI years 0.[5]

This being said, if one should be driven to estimate the lifetime of some newborn infant, thus one will have to consider: This newborn infant is included either among those 36. who die within the first six years; or among those 24 who die between the sixth and sixteenth year; or among the 15 who die between

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[1]This paragraph and the next two are drawn from Book IV, Chapter IV of the *Ars Conjectandi.*

[2]Franz van Schooten 1615-1660. Professor of mathematics, University of Leyden. He taught Christiaan Huygens. Huygens' treatise on probability was appended to his *Exercitationes Mathematicæ* (1657)

[3]Dissertation on Conversion and Reversion of Pronouncements

[4]This publication of Jakob Bernoulli refers to the *Journal des Sçavans* in which was published the mortality table refered to here.

[5]This is John Graunt's table as published in his Observations on the Bills of Mortality, 1662.

the 16th. and 26th. year, or among those 9 who die between the 26th. and 36th. year; or among those 6. who die between the 36th. and 46th. year; or among those 4 who die between the 46th. and 56th. year; or among those 3 who die between the 56th. and 66th. year; or among those two who die between the 66th. and 76th. year; or lastly he will be that one, who dies between the 76th. and 86th. year.

Therefore, there are 36 chances, that he would die within the first six years i.e. that he may survive up to 3 years (for this half is chosen, since on account of the lack of observations not extending themselves into individual years it must be supposed, that anyone is equally likely to die in the individual moments of these six years, and so lest there be a very great danger of straying from the truth, it ought to be presumed, that that man has died in the middle of the time i.e. in the third year); another 24. chances that he would die between the sixth and sixteenth year i.e. that he would probably live up to 11. years (for beyond the six years, which he would pass for certain, he is presumed to live up to 5 years, because this ought to be supposed once again, that in the individual moments of a ten year period, which comes between the sixth and the sixteenth year, one is able to die with equal ease, which similarly ought to be understood in the rest of the following decades); likewise another 15 chances that he would live up to 21 years (if indeed he would die between the 16th & 26th year) another 9 that he would live 31 years. meanwhile another six that 41, four that 51, three that 61, two that 71, one lastly is the chance that he would live 81 years; from which by the general rule having been given in the preceding chapter the expectation of our infant is worth   10.

$$\frac{36 \cdot 3 + 24 \cdot 11 + 15 \cdot 21 + 9 \cdot 31 + 6 \cdot 41 + 4 \cdot 51 + 3 \cdot 61 + 2 \cdot 71 + 1 \cdot 81}{100}$$

$$= \frac{1822}{100} = 18\frac{11}{50} \text{ years}$$

In this same manner it will be found, that of him, who is six years old, life probably will be

$$\frac{24 \cdot 5 + 15 \cdot 15 + 9 \cdot 25 + 6 \cdot 35 + 4 \cdot 45 + 3 \cdot 55 + 2 \cdot 65 + 1 \cdot 75}{64}$$

$$= \frac{1330}{64} = 20\frac{25}{32} \text{ years.}$$

(for from 64 men, who surpass the sixth year, 24. die within the first decade, 15. within the second, 9. within the third, &c.) Equally of him, who is sixteen years, future lifetime is probably

$$\frac{15 \cdot 5 + 9 \cdot 15 + 6 \cdot 25 + 4 \cdot 35 + 3 \cdot 45 + 2 \cdot 55 + 1 \cdot 65}{40} = \frac{810}{40} = 20\frac{1}{4} \text{ years}$$

of him who is twenty-six years old

$$\frac{9 \cdot 5 + 6 \cdot 15 + 4 \cdot 25 + 3 \cdot 35 + 2 \cdot 45 + 1 \cdot 55}{25} = \frac{485}{25} = 19\frac{2}{5} \text{ years}$$

of him who is thirty-six years old

$$\frac{6 \cdot 5 + 4 \cdot 15 + 3 \cdot 25 + 2 \cdot 35 + 1 \cdot 45}{16} = \frac{280}{16} = 17\frac{1}{2} \text{ years}$$

of him who is forty-six years old

$$\frac{4 \cdot 5 + 3 \cdot 15 + 2 \cdot 25 + 1 \cdot 35}{10} = \frac{150}{10} = 15. \text{ years}$$

of him who is fifty-six years old

$$\frac{3 \cdot 5 + 2 \cdot 15 + 1 \cdot 25}{6} = \frac{70}{6} = 11\frac{2}{3} \text{ years}$$

of him who is sixty-six years old

$$\frac{2 \cdot 5 + 1 \cdot 15}{3} = \frac{25}{3} = 8\frac{1}{3} \text{ years}$$

lastly of him who is seventy-six years the probable life is 5 years.

In another way & certainly briefer in reverse order thus we discover these same lifetimes: that he who is 66 years has 2 chances that he would die within ten years, & one, that he would survive to the 76th. year. (in this chance beyond the 10 years which he has in mind certainly he expects 5 years besides) therefore of him the expectation is worth

$$\frac{2 \cdot 5 + 1 \cdot (10 + 5)}{3} = \frac{25}{3} = 8\frac{1}{3} \text{ years}$$

That man who is 56 years, has three chances, that he would die within a decade, three others that he would survive into the state of some 66 years i.e. that beyond 10 years, also he is able to expect up to $8\frac{1}{3}$ years, whence his expectation is

$$\frac{3 \cdot 5 + 3 \cdot (10 + 8\frac{1}{3})}{6} = \frac{70}{6} = 11\frac{2}{3} \text{ years}$$

Thus of that man who is 46 years the expectation is worth

$$\frac{4 \cdot 5 + 6 \cdot (10 + 11\frac{2}{3})}{10} = \frac{150}{10} = 15. \text{ years}$$

of that man who is 36 years

$$\frac{6 \cdot 5 + 16 \cdot (10 + 15)}{16} = \frac{280}{16} = 17\frac{1}{2} \text{ years}$$

11.   of him who is 26

$$\frac{9 \cdot 5 + 16 \cdot (10 + 17\frac{1}{2})}{25} = \frac{485}{25} = 19\frac{2}{5} \text{ years}$$

of him who is 16

$$\frac{15 \cdot 5 + 25 \cdot (10 + 19\frac{2}{5})}{40} = \frac{810}{40} = 20\frac{1}{4} \text{ years}$$

of him who is 6

$$\frac{24 \cdot 5 + 40 \cdot (10 + 20\frac{1}{4})}{64} = \frac{1330}{64} = 20\frac{25}{32} \text{ years}$$

lastly of the newborn infant the expectation is

$$\frac{36 \cdot 3 + 64 \cdot (6 + 20\frac{25}{32})}{100} = \frac{1822}{100} = 18\frac{11}{50} \text{ years.}$$

Also in similar manner the lifetime will be determined of him, who is of an age intermediate among the years 0. 6. 16. 26. &c. e.g. 20 years, for out of 40 16-year-old youths within the next decade 15 die. Therefore, if we should establish that they would be liable to die with equal facility at any single moment of the decade, within four years 6. will die and thus after the twentieth year 34. will survive, from whom again 9. die within the next six years, within the following decade also 9., within the second decade 6., within the third 4., within the fourth 3., within the fifth 2., within the sixth 1, which makes the expectation of some youth 20. years old worth

$$\frac{9 \cdot 3 + 9 \cdot 11 + 6 \cdot 21 + 4 \cdot 31 + 3 \cdot 41 + 2 \cdot 51 + 1 \cdot 61}{34} = \frac{662}{34} = 19\frac{8}{17} \text{ years}$$

Or this way: there are 9 chances that he would die within 6 years, another 25 that he would reach the 26th. year. i.e. that beyond 6. years, which then he is to have for certain, he is also able to hope for $19\frac{2}{5}$ years. this is the probable lifetime of one who is aged 26 years of age, in like manner the expectation is

$$\frac{9 \cdot 3 + 25 \cdot (6 + 19\frac{2}{5})}{34} = \frac{662}{34} = 19\frac{8}{17} \text{ years}$$

as before.

But especially here it must be noted, that, since we say the expectation of an infant or of someone who is 6. 16. &c. years to be worth $18\frac{11}{50}$. $20\frac{25}{32}$. $19\frac{2}{5}$. years &c., these <life expectations> must not be understood thus, that men of a certain age will probably lead lives up to $18\frac{11}{50}$. $20\frac{25}{32}$. $19\frac{2}{5}$. &c. years or that it is equally probable that they have died within this age span rather than beyond, i.e. that among many men of the same age so many would go beyond the specified age as die inside it; but all these <life expectations> ought to be understood according to the age intermediate between the highest & least which men can reach, for these individual years & the two extremes are balanced with 12. one another, i.e. the longest life with the untimely & the anticipated death, it is what the Germans call ein Jahr in daſ ander gerechnet[6] and the French *l'un portant l'autre*,[7] so that it would be contrary to say, that the most probable age of this man & the mean are e.g. 20 years. & this man probably will arrive at his twentieth year.

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[6]One year counting as the other.

[7]Taking one thing with another.

There is a reason for the contradiction, because in the former case also longest lifespan comes into consideration but in the latter it does not, for it is not asked in the question whether someone would have to extend his life to the given year or whether he would surpass the given age by some three, ten, twenty, forty &c. years, but exactly how much is the probability that he may surpass.

Thus the infant according to the former finding will be said to expect $18\frac{11}{50}$ years granted that it must be almost twice more probable that he will not lead his life to that age, for out of one hundred infants scarcely 37 survive after $18\frac{11}{50}$. years. Thus also he who has e.g. two chances to obtain 100, one to obtain 1000, must be said to expect 400, because, although it is twice more probable, that the 100 themselves would occur so much rather than one thousand, in turn even a loss, which he holds as to be feared in this case, is twice less than the gain, which he can hope for in this case.

But if indeed we also wish to determine that time, in which this infant most probably must die, one will need search only within how many years a half part of infants i.e. <when> 50 die out of one hundred, which thus happens: From these 100 36. die within six years. Within the next decade 24. since the number of these now exceeds 50. This must be thus discovered: 24. die within 10 years. Within how many years 14. die (For so many out of those 36 who died in the first six years in order to fill half are wanting) & $5\frac{5}{6}$ years will be found, which added to the first six years will give the sought time period evidently $11\frac{5}{6}$. years.

But not just one mean lifetime is probably able to be determined in this way, but also two, three, or many, i.e. the mean lifetime of greatest duration of two, three, four &c. men of the same or different ages; but before we are able to compute this age the solution of the following problem must be set out first: Given a limit a of years of the longest life within which several men <designated> by the number b are able to die in the individual moments with equal ease, it is sought how many years the longest living probably is able to attain. Thus in respect to $\frac{ba}{b+1}$ years i.e. if there should be 1 person the sought age will be $\frac{1}{2}a$, if 2, $\frac{2}{3}a$, if 3 $\frac{3}{4}a$, if 4. $\frac{4}{5}a$, &c.

13.

For if time a should be divided into innumerable equal parts or moments m, of which number n should be infinite, so that $nm = a$; & one may die living the longest time in the last moment, the remainder will die in the same moment or in some preceding one, & indeed in so many cases, how many nulls, singletons, twos, threes, &c. are contained in the number n, just as those remaining are either 0. or 1. or 2. or 3. &c; indeed

$$\text{either 1. or } n. \text{ or } \frac{n \cdot n + 1}{2}, \text{ or } \frac{n \cdot n + 1 \cdot n + 2}{2 \cdot 3} \text{ \&c}$$

in the cases, from which the product from the number of cases in the number of moments, which he could expect living the longest if he would be supposed to die in the last moment, will be

$$1 \cdot nm, \; n \cdot nm, \; \frac{n \cdot n + 1}{2}nm, \; \frac{n \cdot n + 1 \cdot n + 2}{2 \cdot 3}nm, \; \text{\&c}$$

& the sum of all the products divided by the sum of all the cases i.e. the total expectation of the longest living, who in the individual moments is equally liable to die.

$$\frac{\sum nm}{n}, \quad \frac{\sum n \cdot nm}{\frac{n \cdot n+1}{2}}, \quad \frac{\sum \frac{n \cdot n+1}{2} nm}{\frac{n \cdot n+1 \cdot n+2}{2 \cdot 3}},$$

$$\frac{\sum \frac{n \cdot n+1 \cdot n+2}{2 \cdot 3} nm}{\frac{n \cdot n+1 \cdot n+2 \cdot n+3}{2 \cdot 3 \cdot 4}}, \quad \&c$$

or because $n = \infty$

$$\frac{\int n}{n} m, \quad \frac{\int nn}{\frac{1}{2} nn} m, \quad \frac{\int \frac{1}{2} n^3}{\frac{1}{6} n^3} m, \quad \frac{\int \frac{1}{6} n^4}{\frac{1}{24} n^4} m, \quad \&c$$

$$\text{i.e. } \frac{1}{2} nm, \quad \frac{2}{3} nm, \quad \frac{3}{4} nm, \quad \frac{4}{5} nm, \quad \&c$$

$$= (\text{since } nm = a) \quad \frac{1}{2} a, \quad \frac{2}{3} a, \quad \frac{3}{4} a, \quad \frac{4}{5} a, \quad \&c$$

Thus also one is even able to find the same geometrically; if a curve of such nature should be constructed, so that abscissa x represents time, within which given the men die, the ordinate y would represent the number of chances, in which at the said time they are able to die, the distance of the center of gravity of this curve from the vertex i.e. $\int xy\,dx / \int y\,dx$ will denote the sought number of years (for above we indicate, there to be mutual agreement between the value of the expectation and the center of gravity); hence in our case, where the ordinate is always understood as a power of the abscissa, of which the exponent is less than the number of persons by one, will be (with $y = x^{b-1}$ having been placed)

$$\frac{\int xy\,dx}{\int y\,dx}$$

i.e. the most probable age of the longest living

$$= \frac{\int x^b\,dx}{\int x^{b-1} dx} = \frac{\frac{1}{b+1} x^{b+1}}{\frac{1}{b} x^b} = \frac{bx}{b+1} \text{ or (I put } x = a) \quad \frac{ba}{b+1}. \quad \text{QED}$$

But if now we want to compute the probable lifetime of two newborns A & 14. B e.g. it is manifest that either each of two A & B have died within the first six years (in which case the mean age of the longest living by the method shown will have been shown $\frac{2}{3}$ 6 i.e. 4 years) or A has died between the 6th. & 16th. year & B within 6. years or again B between the 6th. & 16th. year & A within 6. years (in these two cases the age of the longest living is $6 + \frac{1}{2} 10 = 11$ years.) or each of the two A & B have died between the 6th & 16th year (in which case the age of the longest living is $6 + \frac{2}{3} 10 = 12\frac{2}{3}$ years) or A will die between the 16th. & 26th. year & B within 16. years or again B between the 16th. & 26th. year & A within 16 years (in these cases the age of the longest living is $16 + \frac{1}{2} 10 = 21$ years.) or each of two will die between the 16th. & 26th. year

(in this case the age of the longest living is $16 + \frac{2}{3} 10 = 22\frac{2}{3}$ years) & so forth; there are but 36×36 cases that each of two would die in the first six years, 36×24 cases, that A would die between the 6th. & 16th. year & B within 6 years. and just as many that B would die between the 6th. & 16th. year & A within 6 years, 24×24 cases that each of two would die between the 6th. & 16th. year, $\overline{36 + 24} \times 15 = 60 \times 15$ cases that A would die between the 16th. & 26th. year & B within 16. years, just as many that B would die between the 16th. & 26th. year & A within 16. years, 15×15 cases that each of two would die between 16th. & 26th. years &c. and for that reason the expectation of the longest living will be

$= 36 \cdot 36 \cdot 4 + 2 \cdot 36 \cdot 24 \cdot 11 + 24 \cdot 24 \cdot 12\frac{2}{3} + 2 \cdot 60 \cdot 15 \cdot 21 + 15 \cdot 15 \cdot 22\frac{2}{3} + 2 \cdot 75 \cdot 9 \cdot 31 + 9 \cdot 9 \cdot 32\frac{2}{3} + 2 \cdot 84 \cdot 6 \cdot 41 + 6 \cdot 6 \cdot 42\frac{2}{3} + 2 \cdot 90 \cdot 4 \cdot 51 + 4 \cdot 4 \cdot 52\frac{2}{3} + 2 \cdot 94 \cdot 3 \cdot 61 + 3 \cdot 3 \cdot 62\frac{2}{3} + 2 \cdot 97 \cdot 2 \cdot 71 + 2 \cdot 2 \cdot 72\frac{2}{3} + 2 \cdot 99 \cdot 1 \cdot 81 + 1 \cdot 1 \cdot 82\frac{2}{3}$ divided by $100 \cdot 100$. i.e. $\frac{278238}{10000} = 27\frac{4119}{5000}$.

If the probable age of the longest living between two, of whom one is sixteen, the other 46. years <is desired>, it is

$4 \cdot 15 \cdot 6\frac{2}{3} + 9 \cdot 4 \cdot 15 + 15 \cdot 3 \cdot 15 + 3 \cdot 9 \cdot 16\frac{2}{3} + 6 \cdot 7 \cdot 25 + 24 \cdot 2 \cdot 25 + 2 \cdot 6 \cdot 26\frac{2}{3} + 4 \cdot 9 \cdot 35 + 30 \cdot 1 \cdot 35 + 1 \cdot 4 \cdot 36\frac{2}{3} + 3 \cdot 10 \cdot 45 + 2 \cdot 10 \cdot 55 + 1 \cdot 10 \cdot 65$ divided by $40 \cdot 10$ this is

$$= \frac{10191\frac{2}{3}}{400} = 25\frac{23}{48}$$

<div style="margin-left:-6em; float:left">15.</div>

And thus the most probable age not only of two, but also of three, or more men of whatever age is able to be computed, but we omit the more widely extended calculation for the sake of brevity here, <we are> about to reveal still so great a method of discovering by one example, how much would be the probability, that out of two men of different ages one would survive the other. Indeed distinguished Uncle in the annex cited above says that he discovered, that against 59 chances, which deprive a young woman having passed beyond sixteen years of life before an old man of 56 years, there would be 101 chances, in which the contrary happens, and so to such an extent it would be almost twice more probable, that the young woman would survive the old man, than that he the young woman; but the calculation, by which one arrives at this, one has omitted in the same manner, and which we will supply here on that account: Out of 40 youths sixteen years old there will die 15. within the first decade, within the second 9, within the third 6, within the fourth 4, within the fifth 3, within the sixth 2, within the seventh 1. Also out of 6 men, who survive to age 56 years, within first decade 3 die, within the second 2, within the third 1. Therefore to have the expectation of someone of 56 years, i.e. how much might be the probability, that he would survive a youth of 16. years, I set down successively <the probability> that he dies in the first, second, third decade; if he dies in the first, then the youth dies either in the same decade (in which case each of two, since in the single moment they are able to die with equal ease, he possesses half certitude, i.e. neither of two has greater hope of surviving the other) or in the remaining decades (in which case the old man has no degree of probability, but the youth certainly survives the old man); moreover the former is able to happen in 15. chances, the latter in 25. chances,

therefore the expectation of the old man, if he should die in the first decade, is

$$= \frac{15 \cdot \frac{1}{2}c + 25 \cdot 0}{40} = \frac{15}{80}c = \frac{3}{16}.$$

of certitude i.e. there are three grades of probability, of which 16. units make a sure certitude, that the old man would survive the youth; if the old man would be set to die in the second decade, there will be 15. chances that the youth would die in the first decade, and so far that the old man would have a sure certitude, 9. chances that he would die in the second, and the old man would have half certitude, & 16. chances that he would die in the remaining decades, in these cases the old man has no part of certitude, therefore the expectation of the old man now will be

$$\frac{15 \cdot c + 9 \cdot \frac{1}{2}c + 16 \cdot 0}{40} = \frac{39}{80}c.$$

lastly if the old man should die in the third decade, there will be 24 chances, which themselves give sure certitude, if of course the youth should die in the first of two decades, 6. chances, which themselves give half, & 10. chances, which themselves give no part of certitude, whence the expectation of the old man about to die in the third decade is found   16.

$$\frac{24 \cdot c + 6 \cdot \frac{1}{2}c + 10 \cdot 0}{40} = \frac{27}{40}c$$

Since however there are 3 chances that the old man may die in the first, two that in the second, & one that in the third decade, therefore the total expectation of the old man will be

$$= \frac{3 \cdot \frac{3}{16}c + 2 \cdot \frac{39}{80}c + 1 \cdot \frac{27}{40}c}{6} = \frac{177}{480}c = \frac{59}{160} \text{ of certitude}$$

& the $\frac{101}{160}$ of certitude will remain for the expectation of the youth so that the expectations of themselves would hold themselves as 59 to 101 i.e. that out of 160 chances there would be 59 chances, in which it happens that the old man survives the youth, & 101 chances, in which the youth survives the old man; as Uncle clearly discovered.

Of the remainder it must be noted, that by us in this & in the preceding computations (since the predicted observations made above speak only about ages of death from the sixth year, sixteenth, twenty sixth &c.) it must have been calculated, that anyone is subject to death with equal ease in an intermediate time, since this deviates from the truth considerably, I intended to set down more accurate observations to which end from a Friend from our celebrated Swiss city in some manner I took care to be sent to me the ages of almost two thousand men some born in the same year, some having died, but I found beyond every hope, that men of a certain age following these observations come to a much more advanced age than it has been noted in the Chronicles of French

Scholars,[8] and the difference is not at all scant, for I found that the expectation of some infant, & indeed neglecting the fraction, to be 27 years, of someone who is 5 years of age 38 years, who 10 years 37, who 15 years 33, who 20 years 30, who 25 years 27, who 30 years 25, who 35 years 22, who 40 years 20, who 45 years 18, who 50 years 15, who 55 years 12, who 60 years 10, who 65 years 8, who 70 years 7, who 75 years 5, who 80 years 4, who 85 years 3.

17. Since indeed it is doubtful, what would be the explanation of this difference, or that the number of observations made will have been insufficient, for if ages of more men e.g. three, four, ten &c. thousand had been observed, I could have deviated less in truth; or (what rather I would have believed) that in our Switzerland men perhaps because of a more temperate life or better constitution of the air more often would be able to reach the longest lifetime, than in France, where by chance the observations, which are in the Chronicle, will have been made; or rather another thing, in those prior observations we will cling nevertheless to our hypothesis for the time being until we might have better ones, to which end it must be wished for that in one & only one city, especially in one of ours, the Pastores in the White Mountains would record more accurately the ages of their deaths, nor would they neglect this as useless (as it has been done to this degree); for apart from the fact that in the succeedings from the intestate, or other matters, sometimes the doubtful is able to happen, in what year of age someone will have died, there is need of public record keeping for the purpose of coming to a decision, we have this <need> besides of an opportunity from such Bills of mortality, because upon them just as a basis we are able to build a computation of the probability of the age of any man, which computation has the greatest utility in civil life & in our Jurisprudence (as we shall soon see), for henceforth this allows us to determine, how great might be the probability, that someone might attain this or that year, & in the course of a logical consequence at that time an absent man would probably be able to be considered dead, also what might be the fair price in Life Contracts, or rather also when in this contract there may be a place for the remedy of law 2. *C. de rescind. vend.* what may be a realistic rationale for estimating an annual legacy, of food distribution, of usufruct[9] &c. in which manner from such a legacy the Falcidian fourth ought to be subtracted, and many other things, concerning which in order we will go into the following material.

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[8]Johann Bernoulli published shorter papers in the *Journal des Sçavans.*

[9]A usufruct is the right of using the property which belongs to another, and to draw from that property all profit, utility and advantage it may produce, provided its usage does not alter the property. A perfect usufruct is one which can be enjoyed without altering it, but it is allowed that the property may be diminished or deteriorated naturally by time or by its intended use, such as use of a house, a piece of land, or animals. An imperfect or quasi-usufruct is the use of property which would be useless to the usufructuary were it not consumed or expended such as money, grain or liquors.

Chapter 3

CONCERNING THE ABSENT PERSON CONSIDERED AS DEAD

The appellation of absence in our Law, as far as it refers to people, is accustomed to be understood diversely, for he who is absent is referred to either improperly or properly. In the first way through interpretation & assumption of law the mad are considered absent, the insane, the orphaned, drunks, carelessly inactive men, also sometimes the deaf & dumb, granted that they may be present in body *l. 2. § 3 ff. de jur. codicill. l. 27. § 2. ff. de recept. arb. l. 17. § 11. de injur. l. 124. § 1. de R. J. l. 10. quib. ex caus. in poss. eat. l. 1. § 1. de auct. tut. l. 209. l. ult. de V. S.* in the case of some it is accustomed for that adage to 18. be spoken παρὼν ἀποδημεῖ *præsens peregrinatur*,[1] as also they are said not to be at home but abroad, who traveling by plan are not attending to the things which are said.

To this degree it pertains to that of Terence in the Eunuch Act 1 Scene 2.

Cum milite isto præsens absens ut sies.[2]

On the other hand there are those who love absent people as present, just as Virgil <wrote> about Dido in the Aeneid IV:

Sola domo moeret vacua stratisque relictis incubat:
illum absens absentem auditque videtque.[3]

Even more still men dead or absent from this world are said to be absent improperly *c. 7. de offic. deleg. in 6.* Alciatus *ad l. 3. de V. S. n. 11.* who however more correctly are said to have ceased to be, or not to be in the nature of things. *§ 1. l. de inutil. stipul.* Here not unlike absence is that, by which in Civil Law slaves are understood to be absent, because by this they possess no rights of citizenship, whence they are considered dead *l. 32. l. 209. ff. de R. J.*

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[1] While present he is out of town
[2] When you are present with your soldier be as if absent.
[3] Alone she mourns in the empty hall, and falls on the blankets left behind:
though absent, each from each, she hears him and sees him.

In a special sense the name of absence again is accustomed to be brought into use in diverse ways for a diversity of subject matter, as they teach Cujac *l. 7. obs. 35. Goeddeus ad. l. absentem 199 ff. de V. S.* Gail *l. 1. obs. 49 n. 10.* Thus in the matter of rule the one absent is regarded as one who does not possess a place of residence in the same province with the man prescribing. *l. fin. c. de long. temp. præscr.* Schneidewin *ad. tit. Inst. de ususcap. de specieb. præscritp. num. 3.* In contracts and especially in covenants they are said to be absent who for so great an interval of time are away, so that speaking with each other they are unable to hear plainly. *l. 1. ff. de V. O.* In judicial proceedings and declarations he is absent, who is not in a court of justice or else who is not home. *l. 4. § 5. ff. de damn. inf. l. 7. § 4. ff. de oper. nov. nunt.* who is away from the tribunal, or else who lies hidden. *l. 51. § 5. 9. ff. de fideic. libert.* In giving to a procurator the absence is considered the same, if outside the limits there is a master appointing a procurator to an office, for if he was on the grounds, in the market place, in the city and in the adjacent buildings, the procurator of the man present is referred to *l. 5. 6. 7. ff. de procur.* Likewise it holds in establishing a guardian of a minor and curator *l. 173. § 1. l. 199. de V. S.* Goeddeus *ad dd. ll.*

But here we judge him as being absent, of whom one is ignorant of where he is & whether he is. *l. 10. ff. de rit. nupt.* or about whom it is not certain whether he is among the living or whether he has gone to meet death, also of whom the life or fate is uncertain, as they say *Impp. in l. 4. C. de postlim. revers.* the ones absent from one's family our Germans call

die Verſchollenen, von denen man nicht erfahrem kan, ob ſie noch im Leben oder todt ſennd.[4]

19.

Therefore about these absent ones, about whom evidently through many years nothing has been heard or learned, it is sought whether and when they ought to have been considered dead, so that heirs are able to succeed just as if <the absent ones> themselves have died; for because the former, who wishes to enter upon an inheritance, ought to be certain of the death of him, of whom he wishes to be heir. *l. 19. ff. de acquir. vel omitt. hered. § ult. I. de hered. qual. & differ.* and otherwise of him living, of whom death is uncertain, the inheritance is unable to be declared *l. I. ff. de hered. vel act. vend.* but death of this absent kind is able to be demonstrated with difficulty, therefore it suffices to have demonstrated death by conjecture Struve *Exerc. 10. θ 66.* & indeed through long duration of time in which one was absent, the general opinion of lawyers is for the assumption of death to be presumed, who nevertheless in defining that time vary in a marvelous way.

Certain people in the very same way, because this <absent man> was not able to be found with a diligent investigation having been made, for the purpose of record-keeping think him to have departed from human affairs. Baldo after him Gothofredi in *l. etsi certus 6. § si unus 2. ff. de SCto Silaniano.* Mascardi *de probat. concl. 1076. n. 3.* Carpzovius *Ipr. For. part. I. const. 16. defin. 38. n. 3. part. 3. const. 15. def. 57. n. 13.* Mevius *p. 5. dec.* 134. Others

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[4]The vanished, of whom one cannot find, whether they are still alive or deceased.

believe five years to be sufficient for this, thus near to the teaching of Bartolo if a warlike young man, a gambler or dissolute will have begun to wander through the world, & if nothing has been heard or known of him for a period of five years, he ought to be presumed dead. Bartolo *tr. testimon. § mortuum n. 38.* Mascardi *conclus. 1076. n. 5.* Thus in Civil Law the extent of five years time was sufficient for the presuming of death for this, so that the wife of the absent man was able to enter into another marriage, as Julianus[5] the Lawyer stated in *l. uxores 6. ff. de divort. & repud.* on that account indeed according to the Law Code from Emperor Constantine the time has been restricted to a period of four years. *l. 7. C. de repud.*

Christyn teaches that after seven years the absent one is considered dead, and of whom an heir is able to apprehend inheritable assets by private hands & without a judge (because the following French proverb *le mort saisit le vif*[6] *vol. 2. dec. 180. n. 7.* following René Choppin *ad leg. municip. and. lib. 3. de util. and. domin. tit. 2. n. 4.*

Others require an absence of ten years, on that account because a long time is defined in law usually by an interval of ten years Besold. *cons. 167. n. 127. part. 4.* Moreover among the Augustanians in Savoy he is considered dead, who is absent for fifteen years, Faber is a witness in *suo codice lib. 5. tit. 40. def. 3.* Meanwhile others for the sake of presuming death long for the person for a period of twenty years. Klock. *de ærar. lib. 2. c. 48. n. 17.* Thus a unique statute to be stipulated near Nuremberg Johannes Buntz relates *in Dissert. de absente pro mortuo declarando cap. 3.* so that if one will have been absent a period of twenty years, and nothing about that one within this time would have been heard, he ought to be considered dead, and the assets of whom must be surrendered to the nearest blood relations just as the life of the man having died <must be surrendered>. 20.

In the next place there are others who wish to induce an absence of thirty years and not before as a valid presumption of death. Richter. *part. I. decis. 66. n. 6.* where he relates, in the Rule of Bohemia by proclamation the Region of Prague following from a date 29 March 1616 that it has been established, on account that one absent through thirty years, to the year and the day is considered dead, and the assets of him ought to be handed over to the nearest blood relatives without security. It is like that which has been given security by Statute of the Senate of Torgau, by which two <conditions> are demanded (1) that someone has been absent for 30 years, to the year & the day, & (2) he will have completed seventy years, just as it appears from the sense of his statute led by Sigismund Finckelthaus. *obs. 100. num. 10.*

And by a certain general habit of the Germans Besold testifies that it has been accepted *cons. 167. n. 98. part. 4.* that if for an interval of thirty years nothing may be learned of the absent one, he would be considered dead,

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[5]Salvius Julianus worked under Hadrian and the Antonines. He was responsible for the Perpetual Edict of 132. This document consisted of the Prætor's annual edicts and was the rule of ordinary practice. He is the fourth most frequently quoted jurist in the Digest of Justinian.

[6]The dead seize the quick.

although there are some, who demand an even longer time, as of fifty & sixty years, as one is to see in the writings of Besold. loc. cit.

On the contrary a certain man, & among others Finckelthaus. *dict. obs. 100.* determine this time to be one hundred years through *l. 23. § 1. C. de SS. Eccles. l. 56. ff. de ususfr. l. 8. ff. de usu. & usufr. leg.* as if a man is presumed to live so long, which presumption into so much they extend, so that they would place as proof when death is alleged. Alciatus. I. *præs. 44.* Cothmann. *resp. 48. num 4.* Stephen Gratian. *discept. for. tom. 3. c. 562. num 26. & seqq.* And in this fashion the Law Faculty of Leipzig 13 November 1611 responds in the following words:

> Dieweil auß Vermuthung der Rechten der Wiensch biß in hundert Jahr zu leben geachtet wird/ und also auch N. durch die Rechtliche Vermuthung noch im Leben senn kan/ so wird ihm sein geührendes Mutterteil billich außgesezt/ und desselben vollbürtigen Brürtigen auf Security so lange gefolget/ biß ihr seinen Todt/ wie recht/ darthut und erweifet/ von Rechts wegen.[7]

In the same way in the case in marriage the Electoral Assembly 5 July 1609 Finckelthaus writes that they have decided. *d. obs. 100. num. 27.* But they refute this ridiculous enough opinion Carpzovius *part. 3. constit. 15. def. 57.* Richter. *decis. 66. part. I.* they who establish that all these things by the authority of the prudent judge must be left behind, for this man by reason of office will have been able to inquire carefully into, Of what age is that absent man? When did he begin to be absent? For how long would he have been absent? Into what place would he have departed? And if into camps, whether in conflict many had died? If indeed he sailed by sea, whether they who have departed with him would have turned back or not? &c. Menochius *l. 6. præsumt. 49. n. 79.*

21.

But when out of this sole lapse of time of absence it would be legal for the judge to declare him dead, I think it best to be able to define <this> based on observations made concerning the above mortality table, for from this we are able to discover, at which time twice as much, triple as much, quadruple as much, &c. it would be more probable, that one would be dead than as even he would live. Therefore when simply it is said to be probable, on that account remarkably it exceeds half certainty, as above *Chapter* I it has been said, I judge it to be probable enough that this one would be dead, if it would be twice more probable that he is dead than alive, then indeed the probability exceeds half of certainty evidently by a sixth part of certainty, of course. But then it is twice more probable, when so many years are elapsed, that from many men of the same age the number of those, who within these years have died, is twice as great as the number of those, who to this point are surviving. Thus for the

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[7]These people say first through presumption by the lawyers they think the human may live up to one hundred years and thus also the vanished man on account of the presumption of the lawyers yet could be among the living so because of them the proper inheritance is cheaply taken away and the same full-blooded co-inheritors through *Security* so long may follow until you <lawyers> may demonstrate and prove his death in a correct way because that is the law. (Trans. B. Rossa.)

sake of finding out at which time it is twice more probable, that a recently born infant, e.g. may be dead as alive, I search among how many years out of one hundred infants there die 67. So that there remain surviving only 33, which number < 33 > is nearly twice less than that <number 67>, & I arrive at $20\frac{2}{3}$ years; for out of one hundred infants within 16 years there die 60 & in the following decade 15. whence for the purpose of considering the time, within which there die 67, by the Rule of Three thus I say: There die fifteen within 10 years, therefore seven die within $4\frac{2}{3}$ years, which added to 16 years makes $20\frac{2}{3}$. In the same manner for him, who is six years, I find $24\frac{4}{9}$ years, for him who is sixteen 25 years, for him who is twenty-six likewise 25, for him who is thirty-six $23\frac{1}{3}$ years, for him who is forty-six 20 years, the one who is fifty-six 15 years, the one who is sixty-six 10 years, and for him finally who is seventy-six years old, $6\frac{2}{3}$ years.

Because if, e.g. someone therefore will have come to be absent in the twentieth or thirtieth year of his lifetime, & he has been absent for twenty-five years, so that in this fashion within this time nothing would have been heard of him, a judge will have been able to declare him dead, and to concede his assets to the nearest blood relations without security.

I say *without security*: for as in the best way Johannes 𝕭𝖚𝖓𝖘 said in words in the *Dissertatione de Absente pro mortuo declarando cap. 8.* it appears absurd to have by means of security regard for the interests of the absent man, who nevertheless is reckoned dead, and also to favor him, who neglects his own property so much, so that he would seem to consider the <property> as having been abandoned: or to oppress him <the heir> by means of security, to whom the law grants ownership perpetual & irrevocable for the assets of the absent man, just as if he has died, <the heir> who acquires an inheritance in his mind, so that among himself and his own <relatives> it <the assets> ought to remain perpetually. For seeing that someone <namely the absent man> is declared dead, for this very man <the heir> all hope concerning his <the absent man's> return is cut short.

There is not that which someone here would put forward, that by this reason the absent man on the side of his own act is defrauded of his assets, contrary to rule of law, which wishes that which is ours without our act is not able to be transferred to another *l. id quod nostrum II. de R. J.* & against natural justice, which <wishes> the property to be put aside for the master without fault, and also prohibits the other with greater damage & injustice from being made richer *l. 14. ff. de cond. indeb. l. 206. de R. I.*

For it is answered: the loss which someone realizes by his own fault he does not appear to realize l. 203. de R. I. but hardly it is so that he is without fault, who in an interval of so many years towards his own sent forth nothing of letters or of a messenger, so that the defendant ought to have entrusted the whole intention of his inheritance and property and to be regarded as belonging to careless negligence, and to such an extent in a certain way by his own testament to suffer loss, & just as this man, who allows his own property to be acquired by length of use *l. 28, de V. S.*

Or if altogether this matter would seem to have anything of unfairness it

is repaid in public utility as opposed to individuals, for this man is engaged in the same reasoning which is in ownerships acquired by length of use, which on that account has been introduced, so that ownership of property would not any longer be in uncertainty *pr. I. de usucap. l. I. ff. eod.* & so that there may be some end of legal actions *l. ult. ff. pro suo.*

Chapter 4

CONCERNING BUYING EXPECTATION, & IN KIND CONCERNING BUYING ANNUITIES

Although in the Contract of Buying-Selling necessarily merchandise ought to intervene from one side, price from the other *l. I. ff. de rer. permut.* nevertheless some kind of buying is peculiar, which is contracted without property, & which is called the buying of expectation or chance, namely when the future cast of the net by a fisherman, or the driving of game to the hunting nets placed by the hunter, or the panther by the bird catcher, or that which will have been captured by the javelin, or the expectation of inheritance is bought *l. 11. § ult.* 23. *in fin. l. 12. ff. de act. emt. l. 8. § 1. de contrah. emt. l. 7. l. 11. de hered. vel act. vend.*

From this buying the buying of future things has been distinguished, when e.g. produce about to come into existence, the birth of a slave girl, or offspring of livestock are bought; for if the future property will come naturally, as produce, the purchase of the property is understood, if however apparently for such things, or rather those things which by reason of contracting so much are called of the future, the purchased expectation is estimated; in the first case the buying has a condition understood, if anything will have been produced, whence if nothing will have been produced no contract is rescinded; in the second case where clearly the chance event is sold, it is pure[1] & the price is owed, it is legal that nothing will happen. *d. l. 8. pr. & § 1. ff. de contrah. emt. & d. l. 11. § ult. de act. emt.* Schütz. *compend.* Lauterbach. *ad. tit. de contr. emt. pag. 300. 301.* Struve *Exerc. 23. θ. 26. Gomez. resol. tom. 2. c. 2. n. 7.* Doneau. & there Hilliger. *l. 13. c. 1. lit. g.* Molina *de just. tr. 2. disp. 240. n. 14.*

For in such a purchase the merchandise which is sold is the bare[2] & sole expectation, whether it would bring product or not, which expectation is consistent in the law of securing & holding that which will have been gained; therefore if nothing will have been gained the buyer is not able to complain of injury on account of such a deed, which from the beginning is able to hold itself towards profit & loss equally, granted that afterwards it would begin to appear as causing loss, when no one would seem to be injured, when the ratio of the fortuitous

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[1] not suspended by any condition

[2] without condition

chance is equal in both parts. Cravetta. *consil. 598 n. 10.* Conversely if the
gain exceeds the price by a far difference, & the seller wishes to allege an injury
beyond half, it must not be heard, for the purpose of recognizing a just price not
the injury, which happens after the fact, the very time of the contract ought to
be examined. *l. 8. C. de rescind. vend.* Struve. *Exerc. 23. θ'. 86.* Mascardi.
de probat. conclus. 962. num. 4. Mevius. part. 2. dec. 320. Carpzovius.
part. 2. constit. 34. def. 6. Richter. *part. 2. dec. 99. num. 147. & seqq.*
Mynsinger. *4. obs. 73. n. 7.*

    Nevertheless here a remedy ought not to be banished entirely *l. 2. C. de
resc. vend.* as nearly all Masters seem to wish, for although the expectation is
uncertain, just the same it is able to be estimated at a certain value & price
by means of that general rule, which we related above *cap. 1.* E.g. if someone
would buy from another the right of exploiting fishing in a river for a year,
the value of expectation will be determined, if the number of fish having been
captured from several previous years in that river is divided by the number of
years, for the quotient will denote the number of fish, which probably will be
caught in this year, and so a just price in this purchase will be that, which
otherwise must be paid for so many fish; hence altogether it must be said, to
coincide with the benefit of the law *2. C. de resc. vend.* to the buyer if more
than double, or to the seller if less than half of that price will have been paid.
Brunnemann agrees *ad d. l. 2. C. de rescind. vend. num. 19.* who in the
selling of the casting of a net out of fairness decides it must be assisted by the
buyer, if only by error or by false persuasion by the seller the man having been
lead astray will have paid twice more, than another, having knowledge of the
condition of that river, in which fishing has been done, would be about to pay.

    Therefore that which Pufendorf asserts *in elements jurispr. univers. lib.
1. def. 12. § 55.* a fair price in such purchases is that, in which there has
been agreement, it is not true otherwise, than if it is supposed that, the men
contracting in defining a price have provided for that, which ordinarily is accus-
tomed to happen, following that which is stated publicly, future contingencies
are judged according to the common contingency; & this also seems to be the
mind of Pufendorf himself, for thus his words speak themselves:

> So that if there would be agreement in the price of anything by men
> contracting, the unknown amount of which to those men contracting
> would depend upon chance or natural causes, for instance, if someone
> would sell the power of fishing in any river for a price, or the expected
> produce of a garden, of a vineyard, &c. that it will be held for equal,
> on which it has been agreed, although afterwards there will have been
> gain or product either more abundant than that price, or less; since
> when such things would have a remarkable extent, it is customary to
> observe among these, that which ordinarily is accustomed to happen,
> excesses & defects by chance are accounted for, and not out of those
> anything withdraws the stability of the contract.

    Up to this point concerning the purchase of expectation in general, its espe-

24.

cial & even now most customary kind is *Emtio annuorum redituum ad vitam*,[3] by which for a certain price having been paid immediately annual pensions are bought, they must be guaranteed for the days of the lifetime either of the buyer (which happens for the most part), or of the seller or of any third party. Thus the purchased returns are accustomed to be called life-rents, life valuation, Germ. Leibrente / Leibgeding / Leibpfrund / & with body & personality they are extinguished, they neither last beyond, nor transfer to heirs,

> dann Leibrente hei Ben derowegen Leibrente, daß sie nicht länger dann der Leib / dem sie verschrieben / währet / gegeben werden / und also die Obligation mit der Person ein End nimmt /[4]

& just as the usufruct, which is extinguished with the individual. § *finitur 3. I. de usufr.* & personal legal privileges, which do not extend beyond the individual *c. privilegium 7. de R. 1. in 6. l. privilegia 196. ff. cod.* Wehner *in obs. pract. lit. L. voce* Leibgeding.

25.

There are indeed they who deny that such purchase is able to be completed in a legal manner, because it seems to induce the promise <of the payer of the annuity> of longing for death, & it seems to discern impropriety relating to usury, & in appearance Henry of Ghent disapproves of this contract, as usurious & unjust *quodlibeto I. quæst. 39. quodlib. 2. quæst. 25.* which however willingly Guilielmus Bont refutes *tr. de usus. & emtione redit. vital. ac perpet. num. 58. & seqq.* where he reveals life-rents to be constituted justly & legally; & concerning the validity of these rents he is not able to be doubted, Stephen Gratian. reports *discept. forens. c. 756. n. 1. & 2.* They agree in this Du Moulin *de usuris quæst. 72. n. 473.* Rodriguez. *de ann. reditib. lib. 1. qu. 5. num. 5. & seqq.* Sordi. *de aliment. tit. 9. quæst. 11. n. 2. & 3.* Scaccia. *de commerc.* § *I. qu. I. n. 295.* Binsfeld *in c. in civitate. X. de usur. qu. 11.* Decius *consil. 123. n. I.* Boër. *dicis. 44. n. 37. & seqq.* Molina *de I. & I. tr. 2. disp. 388. n. 2. seqq.*

He concurs that concerning Canon Law a doubtful concession has been allowed, through which the Church granted the usufruct to anyone for lifetime on the property by the Church officer. *c. 2.* § *si oeconomus. caus. 10. quæst. 2.* indeed Clement.[5] *I. de rebus Eccl. non alien.* It permits the Prelate, with the necessity or utility of the monastery demanding, for the money of another to yield certain property or the annual rent of the monastery towards the lifetime of that one in annual serfdom; so that there is nothing other, than to grant subsistence to someone. See Du Moulin. *d. disp. 388. n. 3. sub fin.*

Nevertheless there are even among the aforementioned Authorities, they who by such a contract, not even by reason of injustice, but lest the occasion of sin be presented, think it must be withheld, as Sordi d. quæst. 11. n. 30. where he

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[3]The purchase of an annual return on life, i.e. life-rents.

[4]Then a life annuity is called therefore a life annuity, that it is not longer than the life / that it uses up that with which it is associated / and also the obligation ends with the person <when the body dies>. (Trans. B. Rossa.)

[5]Pope Clement V. (b 1264, d. 20 April 1314). He completed the Corpus Juris Canonici with the publication of the Clementinæ.

brings forth the example of Johan Picus,[6] who having stipulated maintenance paid by a great sum of money had been consumed by a certain rich Florentine by poison, because he <the Florentine> impatient by the delay is not able to await the day of natural death.

Truly since the annuity through itself would not allow the opportunity of longing for death, this will have to be reckoned as a fault not due to the contract itself but to the wickedness of the ones forming the contract, for that which does not occur by accident is able to make the contract itself considered illicit; & e.g. from this, because sons would be able to desire the death of parents, so that they would succeed in those goods, the law was not being condemned which established, that sons would succeed parents: thus from this source the annuity contract ought not be condemned. Molina *de I. & I. tr. 2. disp. 388. num. 9.*

Many things which seem to act against the justice of this judgment Linck. brings forth and refutes *dissert. de reditu vitalitio c. 2. n. 13. seqq.* However the chance event alone and uncertainty of human lifetime does not justify the nature of an annuity, as Müller wishes *ad Struv. Exerc. 27. θ. 58. lit. λ. not. 6.* Gail. *lib. 2. obs. 8. n. 2. & 3.* Rodriguez. & Sordi. *locis cit.* Giovanni. Battista Lupus. *in l. 2. C. de pact. inter emt. & vendit. § 2. num. 10.* Wehner *in obs. pract. lit. P. verb.* 𝕻𝖋𝖗𝖚𝖓𝖉.[7] where there he says:

26.

> The contract of purchase of an annual return commonly 𝖊𝖎𝖓 𝕷𝖊𝖎𝖇𝖌𝖊𝖉𝖎𝖓𝖌 has been legal on account of the uncertainty of purchasing life, even if after the fact it would seem excessive, 𝖜𝖆𝖓𝖓 𝖉𝖊𝖗 𝕻𝖋𝖗ü𝖓𝖉𝖊𝖗 𝖟𝖚 𝖑𝖆𝖓𝖌 𝖑𝖊𝖇𝖙 / 𝖔𝖉𝖊𝖗 𝖌𝖆𝖗 𝖇𝖆𝖑𝖉 𝖘𝖙𝖎𝖗𝖇𝖙.[8]

For towards the justice of this the contract is required, in order that an equal condition intercede between the buyer & the seller, & each equally would share in the danger of loss, which is not able to happen, unless the premium would be decided upon following a probable & seemingly true conjecture of lifetime. In order that therefore it would be clear, when such an annuity contract would have been lawful or not, it will be worth the effort to show how one ought to estimate the just price of an annuity; truly in that price of yours to be defined Teachers & customs vary greatly. Du Moulin *tr. de usuris quæst. 72. num. 472.* reports six opinions, the first of Oldradus *quæst. 207.* who wished the fair premium to be the ratio of one to six; second Baldo *consil. 292. lib. 5.* who says the just premium of the purchased life-rent on the lifetime of two to be to the ratio of one to nine; third of the same Baldo *consil. 410. lib. I.* where he decided to value the rent to the ratio of one to ten having been bought on the lifetime of one; fourth again Baldo *consil. 18. lib. I.* where he said to obtain the same in a rent having been bought on the lifetime of two; the fifth of Romanus

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[6]Pico della Mirandola, Count Giovanni 1463-1494. The philosopher Pico spent the last years of his life in Florence and there wrote his most important works, working under the protection of Lorenzo d'Medici. Durant claims that he gave away his fortune as annuities for poor girls. His death at an early age lead to suspicion that he had been poisoned. This is likely untrue.

[7]Prebend. That share of cathedral revenues which goes to the support of a clergyman.

[8]when the beneficiary lives too long. or dies very soon.

consil. 302. who defends the rent of one to eight having been bought on the lifetime of man & wife; sixth Cumanus *consil. 181.* who is of the opinion ten to one year to be the just premium of a purchased rent on the lifetime of a father & two sons, of whom one was 17. years, the other two years old. Of these opinions the first & last Du Moulin rejects *d. queast. 72. num. 480.* the second, fourth & fifth must be understood of men of more advanced age or badly weakened, for the third indeed he wishes to proceed unencumbered on a man of average age; however he understands average to be the age from thirty years to forty in a male, & a little shorter time in a female. *d. quæst. 72. n. 474.*

If the rent should be purchased on the lifetime of two in its entirety, a considerably more ample premium, evidently he believes it must be constituted in the ratio of the twelfth or about *d. num. 480.* unless both would be not of an average but of a little more prone for the worse age and state of health. Gail. *l. 2. obs. 8.* says such contract is just & legal, and the sellers are even bound to respect, if it should happen in the ratio of one annual stipend for ten or twelve 27. by one person of average age, put 30. years old. Guilielmus Bont. *tr. de usur. num. 62.* says that in the valuation common among men the right of receiving 10. in a year for the life of anyone is worth so much, as much as one hundred in denarii at hand at one time. Thus also Frantzk. *resol. l. I. quæst. 2. n. 93. & seq.* determines the justness of the premium in the ratio 1. to 10. on account of this reasoning, because he thinks that the true estimation for the proportion of the purchased rent must be made, as (the words of Frantzk are), just as in the former he has reductions in the premium themselves against the premium of the perpetual return, so also he would have these <reductions>, which happen in this annuity and uncertainty in regard to the redeemables: certainly because just as the perpetual <return> exceeds the redeemable <return> by a third part, & therefore this would exceed the annuity in the same way. And in the Imperial lands he enjoins this to be evident, while the premium of that in the ratio of 5% interest, or if in ratio of one to twenty is constituted, which <the premium> is two parts of the perpetual return commonly in ratio of one to 30. estimated, of which the third part is 10., by which he determines the annuity must be estimated.

On account of this cause even now in Saxony the dowries of noble wives are constituted in ratio of one to ten likewise asserts Frantzk. d. quæst. 2. num. 96. In Spain by the royal ordinance of Philip II from the year 1583. the premium has been estimated in the ratio 1. to 7. by witness Felicano de Solis *de censib. lib. 2. cap. 10.* Rodriguez. *de ann. redit. lib. I. quæst. 5. n. 11.* Guttieriz. *de pract. qq. lib. 2. in calce q. 17.* Covarruvias however *lib. 3. var. resol. c. 9. n. 7.* affirms most frequently it is observed, that those annual returns of yours on lifetime are in ratio one to eight. Say that it deserves to be increased, which we have had before a period of five years in our public New Laws, in England evidently such annuities have been constituted in ratio of one to nine on the lifetime of one, in ratio of one to 11 on the lifetime of two, of one to 12. on the lifetime of three, & in the ratio of one to 15. for the temporary return all the way to 99. years; the written Words of the New Laws of London under the date 7. March 1704. thus they themselves have:

So bald die Königin gestern die Verordnung wegen den Leibrenten gut geheißen / so sennd in 2. Stunden hernach 10000 lb. und heurte 100000 lb. Sterlings eingeschrieben worden; Dieses sennd die Conditionen der Leibrenten: wenn man nemmlich 10. lb Sterlings jährlichen Einkommens haben / und solches biß auf das Leben dreuer nacheinander folgender Personen erstreden wil / so bezahlt man für ein Leben 90. für 2. Leben 110. und für 3. leben nacheinander folgender Personen 120 lb. Sterlings, und wann man 14. lb jährlicher Leibrenten 99. Jahr lang haben wil / so bezahlt mand dafür 210 lb. Sterlings.[9]

28.

But indisputably it is proven that the premium is not so simply able to be constituted short of consideration for the lifetime & health of the buyer, for of these in the definition of the premium of an annuity the best ratio is held, and the same payment ought not be sold for the same premium indiscriminately to a man of any age; this brings about here what Ulpian says about the transaction concerning maintenances in *l. 8. § 10. ff. de transact.* the amount of money, which comes towards the transaction, for the age & health of this man, who transacts, must be arbitrated, because plainly it would be transacted at one time with a boy, at another time with a youth, at another time with an old man. Hence still many reliable men have made a decision concerning this matter, but they wish that the whole matter be left to the prudent decision of a Judge, as Scaccia. *de commerc. § I. qu. 1. n. 267.* where nevertheless he says that, if the buyer should be an octogenarian & infirm, it would be fair to purchase an annual return in the ratio of thirty to one hundred.

But Molina *tr. 2. de just. & jur. disp. 388. num. 5.* advances the rule, how by reason of age a fair premium on the annuity must be evaluated, evidently by attending to the ultimate end, which characteristically & naturally the lifetime of this man is able to reach, into which it is conceded, & in the assumption of half the years, which are from the day, on which he begins the annuity, all the way to that end, thus so that the just premium would be that which in quantity would make equal the payments of all those years: So that if the annuity would be ten ducats annually, & it is conceded towards the life of him that he would be a sexagenarian and he would be expected to be able to live until the age of eighty, the just premium on his annuity would be 100 gold coins, which equals the payments for ten years, and also to that point half the extent, which is from sixty until 80.

A little after around which rule he makes a distinction between personal property & real property, & indeed he does not regularly judge that <the annuity> of the personal property has been lawful, but rather <the return> related to the payment of interest, unless it would happen close to this rule; but when he agrees with an annuity in the case of real property, then a fair premium not close to that rule must be estimated, but he thinks it must be constituted by the authority of a prudent one *d. disc.. 388. n. 8.* But whatever there

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[9]As soon as the queen yesterday approved the ordinance about the life annuity so 2. hours later 10000 lb. and since the beginning 100000 lb. sterling were documented written; these are the conditions of the life annuity: if one namely wants to have 10 lb. sterling yearly income and if this income stretches over the life successively following persons, then for 1 life pay 90, 2 lives 110, 3 lives 120 lb. sterling; and if you want 14 lbs. of yearly income for 99 years, you pay 210 lbs. sterling. (Trans. B. Rossa.)

would be of real or personal property, this rule at least in no way is able to be permitted, for a sexagenarian is likely e.g. to achieve the eightieth year, & thus up to that time he would be able to live twenty years, not yet half of the extent, which is from 60. to 80. i.e. a decade ought probably to be held as his future 29. age, who not equally easily (as here it is tacitly supposed) is subject to death in the individual years of this twenty year period; further although the sexagenarian would be presumed to survive ten years, for that reason nevertheless a fair premium of the annuity ought not to make payments equal to the ten years, but on account of that which is lent, as they say, of immediate payment interest accruing in the meantime ought to be deducted. *arg. l. 45. l. 66. in pr. ff. ad L. Falcid.* for one hundred gold coins are worth more in hand, than ten gold coins to be paid in the individual years of a decade.

Therefore among all these reported opinions which are added without basis, to be sure also we will relate the genuine method of estimating annuities. At first it is well known that, because the principal or the given premium is unable to be discovered, the annual payments ought to exceed ordinary interests, which are paid otherwise towards the same principal, if the property should be redeemable; second it is fair, as that which more than the appropriate interest is paid in individual years, it would be charged to the principal, by which it happens that the principal would be diminished in each year, & finally it would be exhausted entirely, but this, so that an equal condition would happen between the buyer & seller, necessarily ought to happen after so many years, as many as that man, for whose life a return is arranged, is presumed to survive.

Therefore to discover which ratio ought to be between the premium & annual payments, when the annuity after as many years would be extinguished, put the principal or premium$= s$, annual payment$= p$, the number of years, which it is presumed anyone survives, & which previously in Cap. 2 we discover$= n$, and let the ratio of principal to principal increased with interest in the first year be as 1 to m, thus let the principal with the interest in the first year$= sm$, from this if the payment p in the first year should be subtracted from the sum, there remains $sm - p$ for the amount of principal after the first year; hence to discover what would be the principal after the second year, it must be made as 1 to m thus $sm - p$ to $smm - pm$ is the principal increased with interest in the second year, from which again the annual payment ought to be subtracted from the sum, as $smm - pm - p$ is held as the amount of principal, which is after the second year; again in multiplying by m and in subtracting p there is $sm^3 - pmm - pm - p$ as the principal after the third year; similarly the principal after the fourth year will be $sm^4 - pm^3 - pmm - pm - p$;after the fifth $sm^5 - pm^4 - pm^3 - pmm - pm - p$; and after the last or n years $sm^n - pm^{n-1} - pm^{n-2} - pm^{n-3} - \cdots - p$, which because it ought to be$= 0$, it will be $sm^n = p + pm + pmm + pm^3 + \cdots + pm^{n-1} =$(because 30. this series is a Geometric Progression) $(pm^n - p)/(m - 1)$, whence by dividing on each side by $m^n/(m - 1)$, there happens $s \times m - 1 = p - p/m^n$, and in the equation changed to a proportion $p \cdot s :: m - 1 \cdot 1 - 1/m^n ::$(by substituting the value of this $m = 105/100$, & $m - 1 = 5/100 = 1/20$, for today regularly at least 5% is permitted i.e. five per hundred, consequently the ratio of principal to principal with interest in the first year is as 100 to 105 or as 1 to $105/100$)

$1/20 \cdot 1 - 100^n/105 :: 1 \cdot 20 \times 1 - 100^n/105$; which indicates that the ratio of annual payment to premium ought to be 1 to $20 \times 1 - 100^n/105$, which ratio completely will be determined, if next the number of years is substituted for n, which the buyer is expected to live.

Thus if someone wishes to purchase any annuity on the life of a newborn infant, who following the previous discovery probably will live yet $18\frac{11}{50}$ years, the just premium will be in the ratio one to $20 \times 1 - (100 \cdot 18\frac{11}{50})/105$ however the value of this expression is found most easily with help of the Logarithm, for the index $= n$ of whatever number raised to the power of <the index> is held as the Logarithm by multiplying the Logarithm of the very same number by n; therefore with the Logarithm of the fraction $100/105$ having been multiplied it is $-0,0211893$ by $18\frac{11}{50}$ or by $1822/100$ it appears $-0,3860690$ the number of which Logarithm is as near as possible $1000/2433 = (100 \cdot 18\frac{11}{50})/105$, which if next is subtracted from one, & the remainder $1433/2433$ multiplied by 20, it will be held as

$$\frac{28660}{2433} = 11\frac{1897}{2433}$$

for the just premium of the annuity bought on the life of a newborn infant. Similarly by substituting for n $20\frac{25}{32}$ it will be found

$$20 \times 1 - \frac{100 \cdot 20\frac{25}{32}}{105} = 20 \times 1 - \frac{1000}{2576} = \frac{20 \times 1756}{2756} = 12\frac{512}{689}$$

for the premium which ought to be paid according to the law of receiving one annuity on the life of someone, who is six years old. Thus the premium to be paid for the return on the life of someone, who is sixteen years, will be

$$20 \times 1 - \frac{100 \cdot 20\frac{1}{4}}{105} = 20 \times 1 - \frac{1000}{2686} = 12\frac{744}{1343};$$

for the return on the life of someone who is 26 years

$$20 \times 1 - \frac{100 \cdot 19\frac{2}{5}}{105} = 20 \times 1 - \frac{1000}{2577} = 12\frac{616}{2577};$$

on the life of someone who is 36 years

$$20 \times 1 - \frac{100 \cdot 17\frac{1}{2}}{105} = 20 \times 1 - \frac{1000}{2349} = 11\frac{1141}{2349};$$

who is 46 years

$$20 \times 1 - \frac{100 \cdot 15}{105} = 20 \times 1 - \frac{1000}{2079} = 10\frac{790}{2079};$$

who is 56 years

$$20 \times 1 - \frac{100 \cdot 11\frac{2}{3}}{105} = 20 \times 1 - \frac{1000}{1767} = 8\frac{1204}{1767};$$

who is 66 years

$$20 \times 1 - \frac{100 \cdot 8\frac{1}{3}}{105} = 20 \times 1 - \frac{1000}{1502} = 6\frac{514}{1502};$$

who is 76 years

$$20 \times 1 - \frac{100 \cdot 5}{105} = 20 \times 1 - \frac{1000}{1276} = 4\frac{104}{319};$$

likewise for return bought on the lives of two newborn infants, between whom the longest duration of living lifetime (as found above in *Chapter 2*) probably is $27\frac{4119}{5000}$ future years, the just premium will be

$$20 \times 1 - \frac{100 \cdot 27\frac{4119}{5000}}{105} = 20 \times 1 - \frac{1000}{3904} = 14\frac{107}{122};$$

& for the return on two lives, of which one is sixteen and the other 46 years,

$$20 \times 1 - \frac{100 \cdot 25\frac{23}{48}}{105} = 20 \times 1 - \frac{1000}{3466} = 14\frac{398}{1733}.$$

Certainly while I write these things, I perceive that the value of these annuities incorrectly are estimated by supposing the duration of the return to be so many years, as many as someone probably will be presumed to survive; for because the premiums increase not in the same proportion with years, for that reason the just premium of the annuity bought on one life, who within the decade e.g. certainly will die, but in each one of these years of the decade is equally likely to die, ought not be the same as the premium of a temporary return of term five years, granted that the probable life of this man would be five years, but the Arithmetic mean between the single premiums, in which a temporary return is bought of term one, two, three &c. years up to ten. Therefore in order 32. that the true premium of the life annuity would be held, it is necessary to find the premium in individual years, any man is able to survive, and to multiply the same <premiums> by the individual cases of facility, & to divide the sum of all the products by the number of all the cases; for which end I build the following Table, which contains the premiums of temporary returns in individual years from one up to one hundred, where for sake of easier computation I reduce fractions to decimals, so that putting the annual payment= 1.000, the premium of the return to be paid for ten years e.g. would be 7.723 i.e. in ratio of one to $7\frac{723}{1000}$ or 1000 to 7723.

By means of the service of this Table I have found the value & the premium on annuities to be as follows: With an existing annual payment = 1.000, the return of him who is

years	0	6	16	26	36	46	56	66	76
value	9.420	10.600	10.593	10.576	10.164	9.457	8.148	6.545	4.558

Year	Premium	Year	Premium	Year	Premium	Year	Premium	Year	Premium
1.	0.952	21.	12.821	41.	17.294	61.	18.980	81.	19.616
2.	1.868	22.	13.162	42.	17.423	62.	19.029	82.	19.634
3.	2.729	23.	13.490	43.	17.546	63.	19.075	83.	19.651
4.	3.553	24.	13.798	44.	17.663	64.	19.119	84.	19.668
5.	4.326	25.	14.093	45.	17.774	63.	19.161	85.	19.684
6.	5.075	26.	14.376	46.	17.880	66.	19.201	86.	19.699
7.	5.787	27.	14.642	47.	17.981	67.	19.239	87.	19.713
8.	6.459	28.	14.898	48.	18.077	68.	19.275	88.	19.727
9.	7.105	29.	15.141	49.	18.169	69.	19.310	89.	19.740
10.	7.723	30.	15.373	50.	18.256	70.	19.343	90.	19.752
11.	8.304	31.	15.593	51.	18.339	71.	19.374	91.	19.764
12.	8.864	32.	15.803	52.	18.418	72.	19.404	92.	19.775
13.	9.396	33.	16.002	53.	18.493	73.	19.432	93.	19.786
14.	9.899	34.	16.193	54.	18.565	74.	19.459	94.	19.796
15.	10.380	35.	16.374	55.	18.634	75.	19.485	95.	19.806
16.	10.838	36.	16.547	56.	18.699	76.	19.509	96.	19.815
17.	11.274	37.	16.712	57.	18.761	77.	19.533	97.	19.824
18.	11.691	38.	16.868	58.	18.819	78.	19.555	98.	19.832
19.	12.085	39.	17.017	59.	18.876	79.	19.576	99.	19.840
20.	12.461	40.	17.159	60.	18.929	80.	19.596	100.	19.848

Note: This table consists of the values of $20(1 - 1/1.05^n)$. The entries for several years are slightly in error.

33.

    Moreover how the operation will have been devised in said premiums to be discovered, for the sake of brevity we will show by a single example exactly for a youth of sixteen years. From those things which have been said in *Chapter 2* it is established that there are 15 chances, that a youth of 16 years would die in the first decade, 9 that in the second, 6 that in the third, 4 that in the fourth, 3 that in the fifth, 2 that in the sixth, & 1 that in the seventh; if he should die in the first decade, the just premium is 4.558 (for the Arithmetic mean is supposed among the first ten numbers of this Table i.e. among the single premiums for a return of one, two, 3, 4, 5, 6, 7, 8, 9, 10 years, because by hypothesis our youth in the individual years of decade is liable to die equally easily); if he should die in the second decade, the premium 10.519, because likewise the Arithmetic mean is among the premiums for return for 11, 12, 13,... 20 years; similarly if he should die in the third decade, the premium would be 14.179; if the fourth 16.427; if the fifth, 17.806; if the sixth 18.653; and finally if the seventh 19.173; therefore by our general rule the value[10] of this return is

$$= \frac{15 \cdot 4.558 + 9 \cdot 10.519 + 6 \cdot 14.179 + 4 \cdot 16.427 + 3 \cdot 17.806 + 2 \cdot 18.653 + 2 \cdot 19.173}{40}$$

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[10]There is an error in the text. It should read:

$$= \frac{15 \cdot 4.558 + 9 \cdot 10.519 + 6 \cdot 14.179 + 4 \cdot 16.427 + 3 \cdot 17.806 + 2 \cdot 18.653 + 1 \cdot 19.173}{40}.$$

$$= \frac{423.720}{40} = 10.593.$$

Hence it is apparent to what amount our discovered premiums agree with those returns, which of the past age in the seventieth year the Magistrate of Amsterdam for the purpose of holding back the burdens of that fatal war,[11] by which at that time the whole of Belgium was being pressed, he established, witness Casparo Commelino[12] *in Descriptione Urbis Amstelodamensis lib. 6. cap. 56.* 34. *pag. 1205.* where these are his words:

Amsteldam resolveerde, om de onkosten te tragen, tot verdadinge van haar Stad en Burgers, door nieuvve conditien Lijfrenten te negotieren, en lieten de volgende Notificatie uijt gaan: Commissarissen in gevolge en toto voldoeninge van d'opgeleijde Commissie van desen Achtbaren Raad, van date 18. deses (Julii 1672) hebbende ge-examineert op vvat vvijse op het gevoeglykst eenige Geldmiddelen door t'negotieren op Lijfrenten, na proportie van de Jaren uijtgevonden zouden konnen vverden, en dienende den Rade van haar Ed: advijs, zouden vermeijnen dat daarin op de volgende vvyse een Egalisatie, zo na als doenlijk is, zoude konnen vverden geobserveert.

Van				
1 *tot* 20 *Jaren*				1000 *gl.*
20 − 30−				950−
30 − 40−				900−
40 − 45−				850−
45 − 50−				800−
50 − 55−		*Exclusive van*		750−
55 − 60−				675−
60 − 65−				600−
65 − 70−				500−
70 − 75−				400−
75 − 80−en daar boven				300−

100 *Guld. jaarlijkse renten*

Waar op gedelibereert zijnde, is het zelve Concept, zoo als t'leijt, bij den Raad geapprobeert, en zijn Commissarissen voor haar Ed; genome mœijte bedankt; En vvijders de Heeren Thresorieren verzocht en geauthoriseert, om bij provisie met de Negotiatie voort te gaan. Was tetekent D. Schaap[13]

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[11]On 23 March 1672 England declared war on the Dutch republic. On 6 April France did likewise. By 12 June the French had crossed the Rhine. Moving north the armies of England and France had captured Utrecht, the provinces of Overijssel and Gelderland. Only Amsterdam and The Hague remained untaken. The Dutch saved themselves by opening their dikes on these armies. It was in the aftermath of this war that Jan de Witt and his brother were murdered.

[12]Caspar Commelin 1636-1693

[13]Amsterdam resolved, to cover the expenses, to tax her City and Citizens by negotiating on new annuity conditions and she spread the next message: Commissioners have examined and succeeded according to the educated Committee of this respectable Board, on date 18 July 1672, to find ways which would get money available in the next several years by negotiating on annuities and informed the Board of its investigation: advice, it would indicate that in the

And somewhat after I applied this, on the ground that this arrangement, because the buyers of these returns would be of a more advanced age for the most part, & therefore none except a small amount of money would be able to be collected from their premiums, <the arrangement> will have been changed & by Notification on date 18 January 1673 it will have been declared, that those buyers, who would have already surpassed their fiftieth year of age, would pay the premium of 800 florins for 100 annually; although therefore these premiums to us, which we found above, would differ in another manner, if nevertheless the defect <of prescription> of earlier years with excess of the latter, in which the premium is established indiscriminately in ratio of one to eight, since next there ought to be established our computation in ratio unity to precisely 6 or 4, there would be compensation, we will discover among these indeed that no small difference intercedes.

Rather those Annuities differ from these, which twenty years before King Louis XIV of France established, as it is well-known from the Edict concerning this matter publicized in the month of November of the Year 1689 of which these are the principals:

35.

> *LOUIS &c. A cet effet Nous avons fait examiner en nôtre Conseil la proposition qui Nous a été faite, de créer des Rentes Viageres à Fonds perdu, assignées sur le même Fonds que celles de nôtre bonne Ville de Paris, qui ne seroient sujettes à aucunes saisies, même pour nos deniers & affaires, & qui seroient constituées sur un pied proportionné à l'âge des Rentiers, lesquels seroient distribués en differentes Classes, suivant la difference de leur âge; A la charge que la part de ceux qui decederoient accoît aux survivans: En sorte que le dernier vivant de chaque Classe recût seul le revenu entier du capital des Rentes de sa Classe: laquelle proposition &c. A CES CAUSES, & autres à ce Nous mouvans, de l'avis de nôtre certaine science, pleine puissance & autorité Royale, Nous avons Dit, Statué & Ordonné, & par ces Presentes signées de nôtre main, Disons, Statuons & Ordonnons, Voulons & Nous plaît,*

next way an equalization, as good as possible, could be observed.

From				
1 to 20 Years			1000 gl.	
20 − 30−			950−	
30 − 40−			900−	
40 − 45−			850−	
45 − 50−			800−	
50 − 55−	Excluded from		750−	100 Guld. annual pay-
55 − 60−			675−	
60 − 65−			600−	
65 − 70−			500−	
70 − 75−			400−	
75 − 80− and above			300−	

ment.

Having discussed this, the same concept as it seems, is passed by the Board and its Commissioners for their investigation are thanked for the efforts. And furthermore the gentlemen Thresorieren are requested and authorized, with provision, to continue the negotiations.

Signed D. Schaap (Trans. P. Wesseling.)

1. *Que par les Commissaires qui seront par nous deputés, il soit vendu & aliené à nos chers & bien-aimés les Prévôt des Marchands & Echevins de nôtre bonne Ville de Paris, la somme de 1400000. livres actuelles & effectives des Rentes Viageres, à prendre sur tous les deniers provenant de nos droits d'Ayles & Gabelles, & de Cinq Grosses Fermes, que Nous avons Declaré & Declarons specialement & par priviége affectés & hypothequés au payement & continuation desdites Rentes, même par préference à la partie de nôtre Tresor Royal. Voulons que les Constitutions en soient faites par les Prévôt des Marchands & Echevins de nôtre dite Ville de Paris, à ceux de nos Sujets, qui les voudront acuerir &c.*

2. *Qu'aucuns des Acquereurs desdites Rentes viageres venant à deceder, les interêts dont jouïssoient les Acquereurs decedés appartiennent aux survivans de la même Classe par droit d'accroissement, & soïent distribués entre eux d'année en année au sol la livre &c.*

4. *Et pour établir un ordre plus naturel & plus juste parmi ceux qui voudront prendre & lever desdites Rentes, & faire en sorte que chacun se trouve associé avec des personnes à peu prés de son âge, Voulons que tous lesd. Rentiers soïent distribués en quatorze Classes. La 1re des enfans jusques à l'âge de 5. ans accomplis. La 2de de 5. ans jusques à 10. ans. La 3me de 10. ans jusques à 15. ans. La 4me de 15. ans jusques à 20. ans. La 5me de 20. ans jusques à 25. ans. La 6me de 25. ans jusques à 30. ans. La 7me de 30. ans jusques à 35. ans. La 8me de 35. ans jusques à 40. ans. La 9me de 40. ans jusques à 45. ans. La 10me de 45. ans jusques à 50. ans. La 11me de 50. ans jusques à 55. ans. La 12me de 55. ans jusques à 60. ans. La 13me de 60. ans jusques à 65. ans. La 14me & derniere Classe de 65. ans jusques à 70. & au-dessus.*

36.

7. *Que chaque Constitution soit de trois cens livres de Capital, & ne puisse être de plus grosses sommes; mais sera loisible à chaque Rentier de prendre tel nombre qu'il lui plaira de parties de Rentes de trois cens livres de Capital chacune &c.*

8. *Et d'autant qu'il ne seroit pas juste que les Enfans & autres personnes d'un âge robuste, qui selon le cours de nature doivent plus long-tems jouir desd. Rentes, en tirassent un aussi gros interêt que ceux d'un âge plus avancé, les Rentiers des deux premieres Classes jusques 'l'âge de dix ans accomplis, ne seront payés des interˆtes de leur Capital que sur le pied du denier vingt. Ceux de la 3me & 4me Classe de 10. ans à 20. ans sur le pied du denier 18. Ceux de la 5me & 6me Classe de 20. à 30. ans sur le pied du denier 16. Ceux de la 7me & 8me Classe de 30. à 40. ans sur le pied du denier 14. Ceux de la 9me & 10me depuis 40. à 50. ans sur le pied du denier 12. Ceux de la 11me & 12me depuis 50. jusques à 60. ans, sur le pied du denier 10. Et ceux de la 13me & 14me depuis 60. ans &*

au-dessus, à raison du denier 8. &c.[14]

From this article at first the reason is evident, why in these returns a greater premium will have been constituted, than in ordinary annuities, to which thus far we have attended, certainly on account of the law of increasing, in which those rejoice, who outlive those people having died established in the same Class; certainly such a law of increasing brings about nothing different, than that all men ought to be presumed to arrive at the longest age, put 86. years; namely because out of three hundred men approximately, who in the same Class are

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[14]LOUIS &c. To this effect We have had examined in our Council the proposition which has been made by Us, to create some Life Annuity Pensions for Sinking Funds, assigned on the same Funds as those of our good City of Paris, which would not be subjects to any foreclosures, even for our money & affairs, & which would be constituted on a basis commensurate to the age of the Pensioners, who would be distributed into different Classes, according to the difference of their age; On the practice that the share of those who would die accrues to the survivors: In a manner that the last living of each Class receives alone the entire income of principal of the Pensions of its Class: which proposition &c. FOR THESE REASONS, & others to this We moving, on the advice of our Council, & of our certain science, full force & Royal authority, We have Said, Ordained & Ordered, & through those Present signed by our hand, Say, Ordain & Order, Wish & it pleases Us,
1. That by the Commissioners who will be deputized by us, it is sold and yielded to our dear and beloved Provost of Marchands & Echevins (Head of the municipal government of Paris) of our good City of Paris, the sum of 14000000 pounds current and effective of Life Annuity Pensions, to take on all the money proceeding of our laws of Aydes and Gabelles, (Indirect Taxes and Salt Taxes) and of Five Large Firms, which we have Declared and We Declare specially and by assigned privilege and mortgages to the payment and continuation of the said Pensions, as through preference to the part of our Royal Treasury. Wish that the Constitutions of it be made by the Provost of the Marchands & Echivins of our said City of Paris, to those of our Subjects, who will want to acquire them etc.
2. That any of the Purchasers of the said Life Annuity Pensions coming to die, the interest which the deceased Purchasers enjoyed belongs to the survivors of the same Class by rights of accrual, and is distributed among them from year to year to the soil the pound &c.
4. And in order to establish a more natural and just order among those who will want to take and raise some said Pensions, and make in a manner that each is associated with some persons nearer to his age, We wishes that all the aforementioned Pensioners be distributed into fourteen Classes. The first of infants until the age of 5 years completed. The second of 5 years until 10 years. The third of 10 years until 15 years. The fourth of 15 years until 20 years. The fifth from 20 years until 25 years. The sixth from 25 years until 30 years. The seventh from 30 years until 35 years. The eighth from 35 years until 40 years. The ninth from 40 until 45 years. The tenth from 45 years until 50 years. The eleventh from 50 years until 55 years. The twelfth from 55 years until 60 years. The thirteenth from 60 years until 65 years. The fourteenth and last Class from 65 years until 70 & above.
7. That every Constitution is of three hundred pounds of Capital, and is not able to be of greater sums; but it will be permissible to each Pensioner to take such a number that he will please of shares of Pensions of three hundred pounds of Capital each etc.
8. And in as much as it would be just that the Children & other persons of a robust age, who according to the course of nature must enjoy a longer time than the others. Pensions, they likewise draw from it a greater interest than those of a more advanced age, the Pensioners of the first two Classes until the ten year old completed, will be paid interests on their Principal only on the basis of twenty denier [1/240th of a pound]. Those of the third and fourth class of 10 years to 20 years on the basis of 18 denier. Those of the fifth and sixth class of 20 to 30 years on the basis of 16 denier. Those of the seventh and eighth of 30 to 40 years on the basis of 14 denier. Those of the ninth and tenth from 40 to 50 years on the basis of 12 denier. Those of the eleventh and twelfth from 50 until 60 years, on the basis of 10 denier. And those of the thirteenth and fourteenth from 60 years and above, by ratio of 8 denier &c.

arranged, probably one at least will arrive to 86. years, & with one surviving so far the annuities are paid for all, and if thus far every one would survive, therefore with one arriving at this age it is the same, as if all would arrive at that <age>.

Whence it is clear that by the buyers of the first & second Class in each year less is paid than what is just, by constituting for themselves the return in ratio one to 20. For the whole of a temporary return, even if it endures one hundred or one thousand years, for that reason because the principal or given premium is here not able to be returned, is always worth less than redeemable property, which nevertheless regularly is purchased by a premium of twenty to one. Those however who are constituted in the remaining Classes, & are older 37. than ten years, enjoy an exceedingly elegant condition; e.g. a forty year old who is presumed to survive 46. years (indeed he is supposed to arrive at the longest age 86. years) receives one annuity for twelve, while according to our Table a return of 46. years ought to be in ratio of one to $17\frac{880}{1000}$ i.e. in ratio one say to 18. Likewise in all remaining one may observe, with those removed only, who are elderly in the extreme & who are able to expect as much as ten or fewer years, for to those people must be paid a greater payment annually than 1. to 8. certainly one to 7. 6. or 5. &c. as it is proven from the Table. The ratio however because in this arrangement the premiums for the most part would be less, & in regard to the opposite the annual payments greater, than those which are held in our Table, <the ratio> seems to me to be this, because our premiums would have been computed by the measure of 5% interest or five per hundred, where nevertheless in these places, where the commercial use is great, as in France, greater interest would have been permitted, as six, seven or even eight per hundred, so that even if I had substituted these in the place of 5% interest in our Table, in the same proportion, which is observed in the said construction, I would have inscribed <them>.

And there is another kind of life contract, which has a great affinity with annuities, namely that agreement, used most of all today by the Italians, by which the Father of a new born daughter, thus contracts with another, so that with the premium accepted immediately would restore to him four times or five times the amount (which next he grants to the daughter for dowry), if it will have happened, consider that the daughter arrives at the marriageable age put 16. years, but the whole he would retain, if before this age she would die. Therefore in this convention it is sought how much ought to be that, which it is proper to restore in the said time; we therefore determine that: We put the money paid as $= 1$, and that after a year as worth m, i.e. that with interest in the first year to be $= m$. (as above previously too it has been placed) hence that after two years will be worth mm, after three m^3, after four m^4, & after sixteen m^{16}; and so if the daughter indeed should arrive at 16. years, the same one ought to receive m^{16}, but because it is able to happen that she die before this age, therefore it is fair that more than m^{16} be restored, so that this uncertainty again would be balanced with another gain, therefore let us say that what ought to be restored is x, and there will be 40. chances to obtain x & 60. to 0, since there are (as appears from the mortality table recounted above *Chapter 2.*) 40. 38.

chances that she arrive at sixteen years, & 60. that she would die within 16. years, hence her expectation, which in this manner is constructed, is

$$\frac{40 \cdot x + 60 \cdot 0}{100} = \frac{2}{5}x,$$

& because this expectation of her ought to be worth just as much as that which the paid premium is worth after the sixteen years i.e. m^{16}, therefore it will be held $\frac{2}{5}x = m^{16}$, & $x = 5m^{16}/2 =$(put $m = 105/100$ because lawful interest rates are 5%) $5/2 \times 105^{16}/100$, which quantity is held easily by Logarithms, for instance the Logarithm of this $105^{16}/100$ is sixteen times the Logarithm of this $105/100$, therefore with the Logarithm of this number multiplied i.e. $0,021\,1893$. by 16. the Logarithm $0,339\,0288$. results, the number of which is approximately 2.183, which again multiplied by 5/2 gives 5.457, for this which ought to be restored to the daughter after 16. years, & which, as it is apparent it is more than five times the money paid at the beginning.

   For the rest in the last place also this must be noted, that which above concerning remedy *legis 2. C. de rescind. vend.* with respect to purchase of expectation in general we said, furthermore in this convention & in annuities it ought to hold, that there would be a place to rescind, if either the buyer will have given more than double of the fair premium, or the seller will have accepted less than half of the same; unless by chance a suspicion of corrupt usury would follow, then indeed the seller although below half the harm ought to be restored for the whole amount.

Chapter 5

ON THE MANNER OF DEDUCTING THE FALCIDIAN FOURTH FROM A LEGACY OF MAINTENANCES, USUFRUCTS, ANNUITIES &c.

Because in uncertain legacies by reason of chance, as for instance in a usufruct, in payments with respect to legacies for the term of life, in maintenances & the like, it is uncertain whether & by how much the Lex Falcidia[1] would hold a place, therefore various methods are put forth in our laws, by which an heir would be able to save his own fourth; or indeed the legacies do not exceed three-fourths at present, nevertheless there is a suspicion, lest on account of the longer life of the legatee they would surpass three-fourths in the future, & therefore the entire annual payments are paid out for so long a time, until the time will have come, whereby something beyond three-fourths begins to be owed against the Lex Falcidia, in which case all legacies of the individual years are diminished in return *l. 47. ff. ad L. Falcid.* to which end it was necessary for security, *by how much more he will have received that it be restored*, to be interposed in the legacies of this kind by the legatee *l. 1. § 16. eod.* or immediately it appears that there is a place for the Falcidian, & therefore this legacy will be estimated as worth so much, as much as it is able to come *l. 55. eod.* however because this thing itself, how much evidently such a legacy is able to come, is uncertain on account of the uncertainness of the life of the legatee, therefore in l. 68. eod. the rule is proposed, in what way the computation must be made in such a legacy of maintenances, usufruct &c. the text in the Digest of Justinian is quite remarkable, & thus it speaks:

39.

> *Computationi in alimentis faciendæ hanc formam esse Ulpianus scribit, ut a prima ætate usque ad annum vicesimum quantitats alimentorum triginta annorum computetur, ejusque quantitatis Falcidia præstetur: ab annis vero viginti usque ad annum vicesimum quintum, annorum viginti octo: ab annis viginti quinque usque ad*

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[1] The *l. Falcidia* enacted that at least one fourth of the estate or property of the testator should be secured to the legal heirs. The essential feature of this law was that legacies or bequests could not exceed three-quarters of the total estate. Equivalently, a testator could not deprive his legal heir of more than three-fourths of the estate. Legacies in excess of three-fourths were scaled down *pro rata* [according to proportion].

*annos triginta, annorum viginti quinque: ab annis triginta usque
ad annos triginta quinque, annorum viginti duo: ab annis triginta
quinque usque ad annos quadraginta, annorum viginti: ab annis
quadraginta usque ad annos quinquaginta, tot annourm computa-
tio fit, quot ætati ejus ad annum sexagesimum deerunt, remisso uno
anno: ab anno vero quinquagesimo usque ad annum quinquages-
imum quintum, annorum novem: ab annis quinquaginta quinque
usque ad annum sexagesimum, annorum septem: ab annis sexag-
inta cujuscunque ætatis sit, annorum quinque: eoque nos jure uti,
Ulpianus ait, & circa computationem ususfructus faciendam. Soli-
tum est tamen a prima ætate usque ad annum trigesimum, computa-
tionem annorum triginta fieri: ab annis vero triginta, tot annorum
computationem inire, quot ad annum sexagesimum deesse videntur:
nunquam ergo amplius quam triginta annorum computatio initur.*[2]

The words of this law so far are clear, so that I would wonder that G. G. Titius
in observationibus ratiocinantibus in compend. Lauterbach. *observ. 945. pag.
629.* interprets that thus, so that by reason of the age, in which a person is
found to be, it is always proper to subtract a year, in this way evidently, so that
a man newborn is presumed to survive for 30. years, a man of one year hitherto
29. of two years hitherto 28. three hitherto 27. years, & thus further, and that
method of computation might always be observed to the twentieth year; from
the twentieth year truly hitherto 28. <years> from the twenty-first 27. from
the twenty-second 26. years, & thus further, someone might be estimated to
survive.; and thus in the remaining periods of years. For indeed if a computation
in this manner were to be established, a man of 19. years ought to be presumed

40.

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[2]Ulpian writes that this is the formula for a computation to be made concerning mainte-
nance, that:

- from birth to the twentieth year the amount of maintenance is computed as thirty years,
  and the Falcidian <fourth> of this amount is answered for;

- from twenty years indeed up to the twenty-fifth, twenty-eight years;

- from twenty-five years up to thirty years, twenty-five years;

- from thirty years up to thirty-five years, twenty-two years;

- from thirty-five years up to forty years, twenty years;

- from forty years up to fifty years, a computation is made for so many years, as many as
  are wanting in his age compared with the sixtieth year, with one year subtracted;

- from the fiftieth year indeed up to the fifty-fifth year, nine years;

- from fifty-five years up to the sixtieth year, seven years;

- from sixty years to whatever age he might be, five years.

Ulpian states that we make use of this law also in respect to the computation of a usufruct.
Nevertheless it is customary from birth up to the thirtieth year, that a computation of thirty
years be made: from thirty years indeed, so many years, as many as seem to be wanting
compared to the sixtieth year: therefore a computation never more than thirty years is entered
upon. So equally in the case of a legacy of a usufruct to the state, whether simply for the
provision of games or without restriction, the valuation will be of thirty years.

to be alive exactly 11 years while nevertheless another, who survives this man by as much as a single year, would be presumed to survive 28. years, and even more than twice as much; this absurdity certainly is evident all the same, so that Titius ought to apply to himself preferably that which he himself said about others *Incidit in Scyllam qui vult vitare Charybdim.*[3]

And that imaginary absurdity ought not to influence that man, which also Johannes Thomas exposes *de noxia animal. c. 9. n. 9.* on the ground that undoubtedly such a computation might set <the end> before the more remote end of life sometimes, which nevertheless one would not be about to reach, who now is nearer; e.g. a youth of 24. years who is believed to survive hitherto through 28. years, he might arrive at age 52 years, when another of 26. years because he is believed to survive only 25. years, will reach with difficulty the 51st. year. on the other hand a youth of 24. years (as Titius says) certainly even now ought to pass beyond the twenty-sixth year, therefore in regard to that one in retrospect he might arrive at 52. years of this latter one only to 51. years which contradictions are unable to stand simultaneously.

For there is answered in consequence of this and similar objections, which contrary to this computation of Ulpian[4] Titius forms ineptly, a falsity perceived to be attached to our law 68., since it is said, following such a computation the youth of 24. years would be believed to be alive hitherto 28. years & the youth of 26. years hitherto 25. &c. and even as the former would be about to reach 52. years this man the 51st. year. for the law does not say, because it ought to be presumed, but because it might be probable, that those men ought to be alive through so many years, neither does it set before those men the 52nd. year nor the 51st. as the end of life, but this means only that in the computation of the Falcidian fourth the maintenance legacy must be estimated having been bequeathed to the youth e.g. of 24. years toward his life, and if the maintenances had been bequeathed for only 28. years, & this number of 28. years is assumed as the middle among all the ages, which such a legatee is able to reach i.e. because among many youths of 24. years certain <of them> die more speedily, certain others more slowly, and the old ones think that both the heir and the legatee equally in danger of ruin are established, and therefore neither   41. is harmed, if they were to suppose the life of the legatee will endure 28. years thus so that the amount of the maintenances of 28. years would be computed, and of this quantity the Falcidian would be fulfilled; whence nevertheless it does not follow, that the computation, if e.g. there were annual legacies of ten, in the same manner must be established, and also if there were legacies of 280., as Meyer wishes *in Colleg. Argent. ad. tit. ff. ad L. Falcid. θ. 17.* and many others; for from this quantity must be deducted, before the rule of Falcidian law is entered upon as a calculation, the <interest> or profit of average time, which the heirs would have, if it should pay those annual <legacies> of ten successively through *l. 45. pr. l. 66. pr. l. 73. § 4. ff. ad L. Falcid. l. 2.*

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[3]He falls into Scylla who wishes to avoid Charybdis.

[4]Domitius Ulpianus, Roman jurist. Principal writings during reign of Caracalla 211-212. He is the most frequently quoted jurist in the Digest of Justinian with 2462 citations, slightly more than 1/4 total.

§ *2.3. si cui plus quam per l. Falc.* Compare Frantzk. *lib. 1. resol. 2. num. 108. & 109.* hence it is clear the Falcidian must not be deducted from the 280. but only from 149. how much the annual <legacies> of ten paid for 28. years ought to be estimated, namely there is (as it is proved from the Table of the preceding chapter) a return of 28. years the value to its premium in ratio 1 to $14\frac{893}{1000}$ i.e. approximately as 10. to 149.

Besides Titius provokes still another objection yet ridiculous & not much different greatly from the prior, <Titius> who thus wishes to argue: The one who is 20. years is presumed to arrive at 48th. year. therefore in his own time he will reach his 25th. year therefore he will attain his 50th. year for he who is 25. years according to this computation ought to be presumed to arrive at his 50th. year. But this must be noted, that even if we should grant the sense of said law 68 to be, that they who are 20. years & 25. years &c. ought to be presumed to arrive to the years 48. & 50. &c. thence nevertheless it would not follow, that the youth e.g. of 20. years certainly ought to arrive at 25. much less at 48. years, because this 50. years is assigned to the former, who is already (not who finally is believed to be) 25. years however it is uncertain, as he who has 20. years would be about to reach 25th. year. therefore to himself on account of this uncertainty not 50. but fewer e.g. 48. ought to be allotted. Besides if someone therefore, as Titius, should wish to calculate, he would be able to show most easily, that any man ought to be presumed to reach whatever age as you please of one hundred, one thousand, & more years, which not unjustly he ought to count as absurd.

However although such a computation of Ulpian would have nothing of itself from the objections of Titius to be feared, yet this <computation> is not able, much less that of Aemilius Macer,[5] even now to have a place in the Falcidian <which is> to be subtracted, for the number of years which in our Digest of Justinian 68. is assumed for the mean age of each man, & according to which Ulpian says the computation must be made, always is set greater than what is just up to nearly the forty-third year, & afterward less than just, and the mean age of any man does not ever go up to 30. years, but at most to 21. years as it is evident from Chapter 2 where we computed the probability of a human life, indeed not out of the opinion of some Medical Quacks, Physiognomists,[6] Palmists, Augurers, divinists who inspect entrails or similar deceivers of this nature, from anyone of whom Titius guesses that the Romans perhaps have accepted the method of reckoning, which is handed down by this law, but from observations having been made concerning the number of those, who are dead at whatever age.

Therefore by this case of the Falcidian law I think that the ratio is best able to be entered upon as a calculation, if such legacies should be estimated according to the value & premiums of life annuities, just as we determined these things in the preceding chapter. Domat, the French Author maintains a similar opinion au *Traité des Loix civiles dans l'article 8. de la section 2. De la Falcidie.*

42.

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[5]Aemelius Macer, Roman jurist known to have lived during reign of Severus 222-235
[6]Those who judge the character of a person by the features of that person

pag. 496. where among others to said law 68. he elegantly annotates by the following:

> *Mais on ne peut se dispenser de remarquer sur cette loy 68. qui est communement considerée comme la principale regle de cette matiere, que les années des âges y sont sur deux pieds differens, dont on n'en prendroit aucun aujourd'huy pour regle dans l'estimation d'un usufruit ou d'une rente viagere, aprés les calculs qui ont été faits sur les experiences du nombre de personnes qui meurent â chaque âge. Car suivant ces calculs il n'y a que peu d'enfans qui arrivent à l'âge de 30 ans: peu qui de vingt ans aillent à cinquante. Ainsi quand un legataire d'un usufruit n'auroit que quatre ou cinq ans, on n'estimeroit pas son usufruit sur le pied d'une durée de 30 années, & pour cet âge, & pour tous les autres on suivroit plûtôt le pied qui est en usage pour les rentes viageres à fonds perdu. Mais quand il seroit certain qu'un legataire d'un usufruit devroit vivre 30 ans, ou que même un revenu annuel eût été donné à une personne & à ses successeurs pour 30 années, cet usufruit ou ce revenu ne vaudroit pas la somme à laquelle se monteroient ces 30 années, puisqu'une rente perpetuelle ne les vaudroit pas. Ainsi il seroit trés-injuste de regler la Falcidie sur le pied d'une telle estimation, qui feroit qu'un legs d'un usufruit, ou d'une rente viagere de 1000. livres par an seroit estimé plus haut pour la Falcidie qu'un legs d'une rente perpetuelle de pareille somme qui ne vaudroit que 20000. livres &c.*[7]

But so that we would illustrate the matter in an example, imagine that a man having died has left behind 3000. he has bequeathed 800. to Titius 900. to Sempronius & 100. annually to the boy Maevius of six years, I say the legacy of Maevius is worth just as much as 1060. for in the chapter preceding it has been found, that the premium of one thousand annually toward the life of anyone, who is six years old, to be 10600. therefore with 800, 900. & 1060. having been added the sum of all the legacies is held as 2760. from this Falcidian named there 510. must be deducted which more than 2250. three-fourths of

43.

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[7]But one is not able to excuse from remarking on this law 68. which is commonly considered as the principal rule of this matter, that the years of the ages are on two different bases, of which of them one would not take any today for the rule in the valuation of a usufruct or of a life pension, after the calculations which were made on the experiences of a number of persons who dies at every age. For following these calculations there are only a few infants who arrive to the age of 30 years: few who of twenty years go to fifty. Thus when a legatee of a usufruct would have four or five years only, one would not estimate his usufruct on the basis of one enduring for 30 years, & for this age, & for all others one would follow rather the basis which is in the use for the life pensions to sinking funds. But when he would be certain that a legatee of a usufruct should live 30 years, or that similarly a yearly income has been given to a person & to his successors for 30 years, this usufruct or this income would not be worth the sum to which it would ascend these 30 years, since a perpetual pension would not be worth them. Thus it would be very unjust of the Falcidian rule on the basis of such an assessment, which would make that a bequest of a usufruct, or of a life pension of 1000 pounds per year would be valued higher for the Falcidian than a bequest of a perpetual pension of an alike sum which would be worth only 20000 pounds &c

the inheritance evidently are the legacies, whence again, as much as it would be proper to be taken from the individual legacies, it is discovered by the Rule of Combination in this way: 510. ought to be subtracted from 2760. therefore

$$
\left\{\begin{array}{l} 800. \\ 900. \\ 100.\text{ annually} \end{array}\right\} \text{ they ought to subtract } \overset{\text{from}}{\left\{\begin{array}{l} \text{Titius} \\ \text{Sempronius} \\ \text{Maevius annually} \end{array}\right\}} \begin{array}{l} 147\frac{19}{23}[8] \\ 166\frac{7}{23} \\ 18\frac{11}{23} \end{array}
$$

But if Maevius prefers to receive the value of the legacy immediately, subtract $195\frac{20}{23}$[9] from 1060. the heir releases to himself $864\frac{3}{23}$.[10] Hence it is clear what must be realized concerning this manner of deducting the Falcidian, which Titius defines *circa fin. d. obs. 945.* where thus he says:

> *Fac defunctum reliquisse 12000. duobus legasse 6000. tertio 6000. in usumfructum vel alimenta concessisse, quid expeditius erit, quam si heres duobus prioribus 1500. & usufructuario aut alimentario tantundem detrahat?*[11]

I admit, nothing is unencumbered, but also nothing more unjust, for it is evident that too much is subtracted from this, seeing that those 6000. which are bequeathed in the usufructs and maintenances, are not worth just that amount as the legacy of the two previous men, & that the total inheritance therefore has not been exhausted by the legacies, so that nothing would be bequeathed to the heir; for the property which remains with the heir, and the usufruct which will have been returned to the heir with the life of the legatee having been ended, are not of no worth, therefore the heir will charge this benefit in his fourth, & he will subtract so much less from the individual legacies; & indeed in the aforementioned case, if this is the will of the deceased, so that interest on 6000. i.e. 300. would be expended in the individual years in maintenance of the legatee, who would be e.g. six years old, therefore a subtraction must be made: Because 100. annually to the life of someone, who is six years, are worth 1060. therefore 300. annually will be worth three times 1060. i.e. 3180. if you should add 6000. to these (the legacy of the first two) you will have 9180. for the sum of the whole which is the legacy, from which <sum> ought to be subtracted only 180. for the purpose of fulfilling the Falcidian & then from 6000. the heir subtracts $117\frac{11}{17}$ for the first two, & for the maintenance just that amount from the present value, or from the individual annual pension $5\frac{15}{17}$. These are from the method of estimating the annual legacies, usufructs, maintenances &c. & from the reason of subtracting the Falcidian fourth from such legacies.

44.

This observation besides happens, since the valuation ought to be made in this way, not solely in the ratio having been set by the Falcidian law, but also

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[9]In the original 19/23.

[10]In original 4/23. Corrections in all three positions by Nicholas Bernoulli.

[11]Suppose that the deceased has left behind 12000. to two he has bequeathed 6000. to a third 6000. he has yielded to a usufruct or to maintenance, what will be more unencumbered, than if the heir should subtract just that amount 1500. from the two former men & from the usufruct or maintenance just as much.

in disputes of whatever kind, where concerning the presumption of human life even conjecture is urged to be reduced to a certain limit, & in which otherwise very many Teachers establish that a norm must be followed *d. legis 68. ad L. Falcid.*; e.g. if it should be urged concerning a residence to be estimated for life, concerning a lease having been made for the life of a leasee, if someone will have harmed another or will have killed him, & the harmed one or the heirs of the killed would desire the valuation of the ceased services successively, and in other cases. Compare. Sande *lib. 3. tit. 14. def. 14.* Gomez. *variar. resol. tom. 3. c. 3. num. 38.* Covarrubias. *lib. 3. var. resol. cap. 9. num. 8.*

Chapter 6

CONCERNING ASSURANCE & NAVAL INVESTMENTS.

Thus far many questions about the probability of human life presenting themselves out of the Art of Conjecturing might have been able to be settled, but brevity demands, that we exhibit the use of this art now too in others, e.g. in assurance, games, wagers &c. And first indeed assurance offers itself, in which contract how the risk would be able to be estimated, it will be accessible out of the resolution of the following case:

    A seller announces to a merchant, that three ships A, B, C, set sail from port laden with merchandise, & the third indeed C. with 100. bundles (*Bails*) out of which three designated No. 1, No. 2, No. 3 would belong to the merchant, and he would take possession of the merchandise in bundle No. 1 for 1000. No. 2 for 2000. No. 3 for 2400 florins. For some time afterwards it is related to that one, that one of those three ships has perished in a shipwreck, and none except 20 bundles have been rescued on the waves. The slightly more timid merchant impatient from the uncertainty, whether the loss will have affected himself also, prefers that it remain for himself to sell for the expectation of another, than to hover between hope and dread too long. It is sought how much must be estimated, on the ground that one would be able to anticipate fairly & rationally. Answer 3960 florins.   45.

    For if that ship, which has endured shipwreck, should have been the third C, which would bear the merchandise of our merchant, the merchant would be able to expect, because out of 100 bundles precisely 20 i.e. the fifth part have been saved, none except a fifth part of his own merchandise, that is 1080. because now ill fortune, if one number of the ships should be observed (another must have been designated, if one of these three in comparison with the others should have been exceeded by dry rot and old age, by sails and yardarms badly equipped, by each new captain and inexpertly instructed &c.)[1] <ill fortune> will have been able to happen to each one of the three $A, B, \& C$ equally easily, therefore there are two cases so that all the merchandise would have been saved,

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[1]This phrasing is from Book IV, Chapter 2 Axiom 2 of the *Ars Conjectandi*.

one that a fifth part only, and so the expectation of the merchant is worth

$$\frac{2 \cdot 5400 + 1 \cdot 1080}{3} = \frac{11880}{3} = 3960.$$

as said.

In like manner the risk is able to be estimated, that the ship will undergo, which not yet has set sail from port, if out of many ships, which will have made the same passage, the number of these would be observed, which will have reached the destination place unharmed. E.g. if more frequently it should have been discovered, that out of one hundred none but 90 of the ships have escaped unharmed, the risk will be estimated as a tenth part of the value, because that one ship is worth one with the merchandise it contained.

Hence also it is clear, how the amount of interest in naval investments ought to be determined, because, as Grotius notes rightly *de I. B. & P. l. 2. c. 12 § 5.* what has been mixed out of the contract of mutual & averted risk, namely at what price the risk ought to be estimated, which the creditor sustains of money of passage, by so much the nautical interests ought to surpass ordinary interests, which consequently are able to be greater or lesser than one percent, according to the ratio of greater or lesser risk; however so that we would set the limits generally, by how much the interest in single months would have been discharged, let the lot or the amount of money of passage be put$= a$, the ordinary monthly interest[2] $= b$, the nautical interest$= x$, number of months, in which naval interests run i.e. in which the ship navigates$= n$, the number of cases in which the ship safely comes into port$= p$, & the number of cases in which the contrary happens$= q$, or since out of $p + q$ ships the number of them which would have been saved $= p$, & the number of them which are lost$= q$. Since therefore there are p cases, that the creditor receives the total lot a together with the naval interests, which in the individual months are x, and so in n months they make nx, & q cases that he would receive nothing, therefore the expectation of him will be

$$= \frac{p(a + nx) + q \cdot 0}{p + q} = \frac{pa + pnx}{p + q},$$

46.

however this expectation ought be worth just as much as that amount, which the creditor would have been about to receive, if he would have lent his money at ordinary interest i.e. $a + nb$, hence it will be $(pa + pnx)/(p + q) = a + nb$, or $pa + pnx = pa + qa + pnb + qnb$, & I take away from both sides pa, $pnx = qa + pnb + qnb$, whence next I divide by pn it will produce $x = (qa + pnb + qnb)/pn$.

For if already a voyage should last e.g. three months, & the number of cases $p \& q$ would have themselves as 9 & 1. (i.e. if it should have been observed, that out of 100 ships none but 90. or out of 10. one has reached the place of destination) and the principal or loan of money would be $a = 1200$. & the monthly interest of this principal was $b = 5$ (for there are annual ordinary interests as 5 per 100 & monthly 5 per 1200.), with the values having been

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[2]The interest is the actual amount, not the rate.

substituted into the equation just now discovered for the corresponding letters themselves, sought would be the amount of naval interest to be paid in the individual months or $x = 1350/27 = 50$ which as it is evident is much greater than one percent, as it is a twenty-fourth part of the principal. If n should be= 5, $p = 35$, $q = 1$, the interest would be= 12. and indeed precisely one percent.

For according to assurance commonly that contract is returned, in which for some premium someone restores the secure life of another, therefore so that, if he will have died before the determined the time, the assurer would pay a certain sum of money; concerning this contract see Molina. *de I. & I. tr. 2. disp. 507. num 12.* Scaccia. *de commerc.* § *1. q. 1. n. 128. 133. & 142.* where he shows a Rule of Thumb or the formula for the assurance on human life, to last to a certain time.

There are certainly those who deny that such a contract is permitted, & for the sake of it they bring forth the following reasons, that human beings are not contained in the name of merchandise. *l. 207. de V. S.* that there would not be a buyer and seller of a free man *l. 6. in pr. l. 34. § 2. l. 70. de contrah. emt.* that a free corpse would not receive a valuation *l. 3. si quadr. paup. fec. dic. l. 1. § 5. de his qui effud. vel dejec.* But someone does not see, that in vain these laws are opposed, since in this contract no man nor free corpse, but only the probability of human life would come to valuation; for so that the contract would be just, nothing else is required, than as gain, which the assurer has, if the man concerning whom it is urged should reach the determined age, to the loss which he suffers, if he should die before his lifetime, he would hold himself reciprocally as the cases of ease in which this latter is able to happen, to the cases in which the former is able to happen: whence easily by the Rule 47. of Three it is possible to discover, how much there ought to be, what more than that which credits the assurer which it would be just to restore, if it will have happened, that the person dies later than the prescribed age.

E.g. if the assurer will have promised, that any newborn infant is about to prolong its life beyond 16. years, and a premium has been accepted together with interest of 16. years, which the assurer gains, if it should happen, that the infant outlive sixteen years, it would be worth 3000. it will need to be said: as 60. is to 40. (for, as from observation, which above in *Chapter 2* we have considered, it is evident, among 100. infants there are 60. who die below sixteen years) therefore 3000. is to 2000. which the assurer more than that which now he accepts in assurance will be held to pay, if the infant will have been dead within 16. years.

Chapter 7

CONCERNING GAMES, WAGERS & LOTTERIES

Games and Wagers have been lawful, provided that they would be moderate, & they would be made from honorable matters, & the one side would be equal to the other, i.e. the risk of losing and the hope of gaining on both sides would have a proportion to the event, about which there is contention; e.g. in a game which on the dexterity of the body or nature is fixed, if the dexterity of one will have exceeded the dexterity of the other by twice as much, there is equality, when here also he <the other> would put down the double premium; or if there should be ten, of whom one at a time they would set down a gold coin, and also they would compete with this agreement, so that he who will have thrown the most, he would carry off all <of the ten>, indeed it may seem unfair, when he who experienced a risk of only one gold coin, may gain nine, but at the same time it must be considered, here the risk of losing is nine-fold greater, than the hope of winning. Pufendorf. *de I. N. & G. l. 5. c. 9. § 7.*

Hence it is evident how in this matter the Art of Conjecture would be necessary, for when the conditions of those gaming or competing would be able to vary completely & to be subject to innumerable cases, on that account no one without the assistance of this Art will have been able to discover easily the number of cases, which would favor or oppose this or that fellow gambler or guarantor of the wager, and thereupon to determine, whether each of the two   48. equally must be determined in the condition of winning & losing, or not. Because in order that more be revealed, we shall mention an example in wagers, for in what manner in a game the lots or the expectations of fellow-gamblers would be able to be defined, copiously Huygens in the unique pamphlet *de ratiociniis in aleæludo*, & my famous Uncle in the entire third part of his *Tractate on the Art of Conjecture* have revealed.

Quite famous today are those Wagers of the Genoise, which are established publicly on the occasion of the elections, which happen in the individual years in Genoa, when five out of one hundred Senators are selected by lot, who in this year discharge the more principal offices; therefore then the wealthy merchants are accustomed at this certain time, before the decision by a lottery would occur, to enter upon a contest with others in this law, in order that whoever will have been willing to compete, he may set out a token for as much as he

wishes, & it would name five of these one hundred <Senators>, & if afterwards the lottery will have produced, that one of the named will have been elected, he would be about to receive a certain sum of money, upon which it has been agreed; if two will have been elected, a greater <sum>; if three, an even greater <sum>; if four, again a greater <sum>; if five, again a greater <sum>; if indeed none of the nominated will have been elected, he would make an expense of the losing token; it is sought, for how much the prize in each chance ought to be established, in order that an equality would be strived for in the lots?

Let the token or price to be expended $= a$, the prize has been established or rather must be established, if one of the nominated is selected$= t$, if two $= u$, if three $= x$, if four $= y$, if five $= z$; now for the sake of brevity the number of cases or of lots would be set, in which five are being selected $= b$, in which four $= c$, in which three$= d$, in which two$= e$, in which one$= f$, in which none$= g$. Therefore there are b cases with the result that someone would acquire z, c cases that y, d cases that x, e cases that u, f cases that t, & g that none, and for that reason the expectation will be worth

$$\frac{b \cdot z + c \cdot y + d \cdot x + e \cdot u + f \cdot t + g \cdot 0}{b + c + d + e + f + g}$$

which ought to be equal to the price paid a, whence the equation will be held as $bz + cy + dx + eu + ft = ba + ca + da + ea + fa + ga$, therefore with the problem it would be indeterminate, indeed four are able to be chosen at will, certainly because there is a combination in the nature of the thing, as the prizes are reciprocally proportional to the number of chances, i.e. for which reason as the chances are fewer, those prizes would be presented as more prized, hence in the place of z, y, &c. we place a single unknown z only, with the remaining existing having been made proportionate to that one $bz/c, bz/d, bz/e$ & bz/f, whence the equation would be of such kind

49.

$$5bz = ba + ca + da + ea + fa + ga \quad \text{or} \quad z = \frac{ba + ca + da + ea + fa + ga}{5b}$$

or (if for the number of all the cases $b + c + d + e + f + g$ let h be put)

$$z = \frac{ha}{5b}, \quad \text{whence} \quad y = \frac{bz}{c} = \frac{ha}{5c}, \quad x = \frac{bz}{d} = \frac{ha}{5d}, \quad u = \frac{bz}{e} = \frac{ha}{5e}$$

& finally

$$t = \frac{bz}{f} = \frac{ha}{5f}.$$

If now there is a token which ought to be set down or a would be one gold coin, & for the letters b, c, d, &c. the values or themselves would be substituted, these which are found by means of the noted combination rules, indeed

$b = 1, \quad c = \frac{5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4} \times \frac{95}{1} = 475, \quad d = \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} \times \frac{95 \cdot 94}{1 \cdot 2} = 44650,$

$e = \frac{5 \cdot 4}{1 \cdot 2} \times \frac{95 \cdot 94 \cdot 93}{1 \cdot 2 \cdot 3} = 1384150, \quad f = \frac{5}{1} \times \frac{95 \cdot 94 \cdot 93 \cdot 92}{1 \cdot 2 \cdot 3 \cdot 4} = 15917725,$

$g = \frac{95 \cdot 94 \cdot 93 \cdot 92 \cdot 91}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 57940519, \quad h = $ the sum of the preceding numbers

$= \frac{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 75287520.$ the prizes are found $z = 15057504$ gold.

$y = 31700\frac{4}{475}$ gold, $x = 337\frac{6227}{22325}$ gold, $u = 10\frac{608002}{692075}$ gold, $t = \frac{15057504}{15917725}$ of one gold coin.

Hence it is evident how much the Genoise merchants would engage in fraud, while for one gold coin regularly they promise only 10,000 gold coins, if five, 1500. if four, 300 if three, 10 if two, & one if one of the named will have been elected, for granted that in this last case those, who contend with the merchants in the way named, would have something of a profit, nevertheless by much more there is a loss, which they suffer by the remaining four chances, which clearly is evident, if we should seek the expectation ourselves, but that is

$$= \frac{1 \cdot 10000 + 475 \cdot 1500 + 44650 \cdot 300 + 1384150 \cdot 10 + 15917725 \cdot 1 + 57940519 \cdot 0}{75287520}$$

$$= \frac{43876725}{75287520} = \frac{2925115}{5019168}$$

of the one gold coin, but they ought to expect so much, as much as they have set down, i.e. one gold coin, therefore the Genoise dealers defraud the individual gamblers the 2094053/5019168 parts of one gold coin, from which it is appropriate, not least a Ministry is able to permit such gambling, seeing that <it is> completely unjust, by public authority, & the merchants are held to the restitution of it, which they have received as more than just.

Indeed unjust are these Genoise wagers, which Juan Caramuel[1] calls by the proper name *Concertationes Cosmopolitanas*, also he asserts in *Mathesi Nova, syntagm. 7* but nevertheless the prizes, which he assigned to the single cases, still in truth fall short, moreover his errors & paralogisms, which here and there he commits in the expectations to be defined of the players, it would be excessively obliging to show in this.

We would add rather something about the Jars of Fortune of Lotteries, thus in the words of the Belgian voice *Loten*, Latin *sortiri*, of which the use today is most frequent. But the jar of fortune is set up in this way: With a certain number of tickets having been thrown down into an urn, of inscribed and of empty <tickets>, a chance is returned for the price of removing the same, thus so that this one extracting would receive, what the inscription of those exhibits on themselves; it is demanded for the justice of it, that the value of all the tickets chosen successively would not greatly exceed the value of the things having been set out there: for because also the costs must be made, & such jars are generally used towards the collection of money to be expended upon public works, or also

50.

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[1] Juan Caramuel 1606-1682. was a Cistercian and author of some seventy works. He is most famous for the work *Mathesis biceps: Vetus et nova* [Two-headed mathematics: old and new]. Bernoulli, in a letter to Montmort (p. 387), mentions Caramuel. "A Jesuit named Caramuel, who I mentioned in my Thesis, wished to push these matters, and even criticize M. Huygens in the Treatise that he names KYBEIA [Dicing], and what he has inferred in his great Works of Mathematics; but as all this that he gives is only a heap of faulty reasonings, I reckon it as nothing."

the alleviation of others in need, on that account from exact equality a portion is held back, to such an extent so that such a difference, by which commonly the value of all the tickets together exceeds the prizes having been put forth, would have a reason of a certain voluntary tax, or of the alms having been enticed in a merry way, as well the esteemed Pufendorf speaks *loc. supra cit.*

Such a jar of fortune, being composed out of pure annuities, not long ago in Belgium has been defined under the fairest conditions, of which we have a description in the New Laws of Bern, in which on the date Amsterdam 15 March 1709 these words are held as having been written in the French language:

51.

> *Voici le plan de la Loterie de Rentes Viagéres, qu'on propose de faire en ce Païs, & qui a été envoyé aux Villes, pour avoir leur approbation. Il y en aura 8000. Billets à 250. Florins chacun, ce qui monte à 2000000. De ces 8000. Billets il y en 1300. Noirs ou Prix & 6700. Blancs. Ces derniers porteront 6. pour cent d'interêt pendant la vie. Cet interêt, de même que les Prix, sera exempt du 100ᵉ & 200ᵉ denier & de toutes autres Charges. Les Prix seront partagez de la maniere suivante. Deux Lots à 3000. Florins de Rente chacun, faisant 6000. Quatre L. à 2000. fl. de R. ft. 8000. Quatre L. à 1000 fl. d. R. ft. 4000. Huit L. à 500. fl. d. R. ft. 4000. Quatorze L. à 250 fl. de R. ft. 3500. Trente L. à 150. fl. d. R. ft. 4500. Trente L. à 100 fl. d. R. ft. 3000. Douze cens huit L. à 30. fl. de R. ft. 36240. Ce qui fait 1300. Lots, & 69240. florins de Rentes Viagéres. Les 6700. Billets Blancs à 6. pour cent produisent 100500. florins de Rentes Viagéres. Le premier & le dernier Billet chacun de 150. florins de Rentes Viagéres. Ce qui revient à 8000. Billets de 250. florins chacun, faisant 2000000. de Florins en fond ou Capital, produisant par an 170040. Florins de Rentes Viagéres. Ceux, qui voudront convertir leurs Rentes Viagéres en Obligations à 4. pour cent sur l'Etat, pourront le faire en tout ou en partie, de sorte que le Prix de 3000. Florins de Rentes Viagéres pourra être changé pour la somme de 35250. fl. & les autres prix à proportion. On conte de tirer cette Lotterie le 1. de Mai prochain, & 6. semaines aprés on sera obligé de donner les noms de ceux, qu'on voudra mettre dans les lettres de Rentes Viagéres. On pourra les mettre sur plusieurs Têtes, mais pas moins de 100. florins de rente sur chacune.*[2]

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[2]Here is the plan of the Lottery of the Life Annuity Pensions, that one proposes to make in this Country, & which has been sent to the Cities, in order to have their approbation. There are 8000 gold Tickets at 250 Florins each, that which amounts to 2000000. Of these 8000 Tickets there are 1300 Black or Prize & 6700 White. These latter will carry 6% interest during life. This interest, just as the prize, will be exempt from the 100th & 200th tax & all other Charges. The Prize will be shared in the following manner. Two Lots at 3000 Florins of Pension each, making 6000. Four Lots at 2000 fl. of P. making 8000. Four L. at 1000 fl. of P. making 4000. Eight L. at 500 fl. of P. making 4000. Fourteen L. at 250 fl. of P. making 3500. Thirty L. at 150 fl. of P. making 4500. Thirty L. at 100 fl. of P making 3000. Twelve hundred eight L. at 30 fl. of P. making 36240. This which makes 1300 Lots & 69240 fl. of

From these words it is clear, that the value of all the tickets taken together is 2,000,000 florins, according to which in the individual years 170,040 florins are paid in the annuities, but because someone, if he wishes, is able to change annuities into redeemables, with the result that for the return of 3,000 florins he would receive a lot of 35,250 florins, & from this sum in the individual years I put 4% interest, i.e. 1410 florins, and also to such an extent the annuity of 3000 florins would be estimated to be worth just as much as 35250 florins, therefore 170,040 florins yearly will be worth just as much as 1,997,970 florins however much therefore the value of all the tickets <will be worth> the value of all the payments taken together, that is 1,997,970 florins, it would exceed by 2030 florins, & the annuity of one florin yearly would be put to be worth $11\frac{3}{4}$ florins (for it is $3000 \cdot 35250 :: 1 \cdot 11\frac{3}{4}$) when the value of it, as we found above in *Chapter 4*, at most would be in the ratio of one to $10\frac{600}{1000}$ or $10\frac{3}{5}$, abundantly however such a difference is balanced with this interest, evidently seeing that such annuities would be exempt from the tax of 1% or 2% coin, and from other taxes; add that, if in d. cap. 4. where for the interest we have set down 5%, the annuities would have been computed on the standard of interest of 3 or 4% (as this ought to happen) a richer premium would have been produced than $10\frac{3}{5}$ for 1 whence it is clear, how much more popular this lottery would be, than the others which commonly are accustomed to be established, where one who brings forth a white or blank ticket, entirely because he exposed silver, one certainly <who brings forth> a black <ticket>, regularly loses a tenth part of his ticket, since in this lottery that man, who draws an empty ticket, would have in single years 6%, this is, for 250 Dutch florins or 100 Imperial thalers, six Imperials, which if they should be multiplied by $11\frac{3}{4}$ are discovered to be worth $70\frac{1}{2}$ Imperials, accordingly this man, who extracts a white ticket, out of 100 Imperials, for which he has purchased the ticket, loses nonetheless $29\frac{1}{2}$ Imperials.

52.

Life Annuity Pensions. The 6700 White Tickets at 6% produce 100500 florins of Life Annuity Pensions. The first and last Ticket each of 150 florins of Life Annuity Pensions. That which returns at 8000 Tickets of 250 florins each, making 2000000 of Florins in fund or Principal, producing per year 170040 Florins of Life Annuity Pensions. Those, who will wish to convert their Life Annuity Pensions into obligations at 4% on the State, will be able to make in all or in part, of the kind that the Prize of 3000 Florins of Life Annuity Pensions can be changed into the sum of 35250 fl. & the other prizes in proportion. One relates to draw this Lottery on 1 May next, & 6 weeks after one will be obliged to give the names of those, who one wishes to place in the letters of the Life Annuity Pensions. One will be able to put on many Heads, but no less than 100 florins of pension on each.

Chapter 8

CONCERNING A PREGNANT WOMAN, HOW MANY INFANTS PROBABLY IS SHE ABOUT TO BEAR
ad l. 3. ff. si pars hered. petat.

It is sought if a man having died will have left behind one son & pregnant wife, what portion of the inheritance the son would be able to seek, because it is uncertain, whether one, or two, or many would be about to be born; here if it should be answered following the Roman Laws, it must be stated, that the son is able to seek only the fourth part; since namely five infants would be able to be born[1] *l. 36. ff. de solut.* & yet usually one would be accustomed to be produced in one birth, on that account the Roman Lawyers holding to the middle between one & five have proposed, that three fetuses are enclosed in a mother's womb; these three therefore, because of their advantage, with the inheritance evidently to be acquired, it is considered, they are held as now having been born *l. 7. l. 26. de stat. hom.* and from this cause the son meanwhile is heir only of a fourth part through *l. 3. si pars. hered. pet.* where likewise Lawyer Paulus[2] replies:

> *Prudentissime juris auctores medietatem quandam secuti sunt, ut quod fieri non raro admodum potest intucrentur, id est, quia fieri poterat, ut tergemini nascerentur, quartam superstiti filio adsignaverint,* τὸ γὰρ ἄπαξ ἤ δις, *id est, quod autem semel aut bis existit (ut ait Theophrastus)* παραβάινουσιν ὁι νομοθέται, *id est, prætereunt legislatores. Ideoque & si unum paritura sit, non ex parte dimidia, sed ex quarta interim heres erit.* addendum l. 28 § fin. de judic.[3]

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[1] The Roman jurist, Laelius Felix reported that an Alexandrian woman, Serapia by name, was brought before the Emperor Hadrian with 5 children. Four were born at the same time. The fifth was born 40 days later. The jurist Gaius repeats the story but without the interval between births.

[2] Julius Paulus worked during the reigns of Septimus Severus and Caracalla. He was exiled under Elegabulus and recalled by Alexander Severus. He held the office of Praetorian Prefect. Paulus is the second most frequently quoted Roman jurist in the Digest of Justinian with 2083 or 2080 separate citations, slightly more than 1/5 total.

[3] Most prudently the law authorities have sought a certain mean, in order that what is able to happen not quite rarely they would contemplate, that is, because it had been able to happen, that triplets would have been born, they will have assigned the fourth part to the surviving son, *for this which is once rather than twice*, that is, for that which once or twice

Truly I do not see, how this regular arrangement would accord with these words of Theophrastus, which in this law itself are appealed to: *What once or twice appears, legislators disregard*, since it happens most rarely, that from one
53.
birth three, much less would more be produced; moreover not rightly *authors of the law have sought the mean* by the choosing of the number three for the number of infants having been born, for if out of several thousand women, who have gestated a fetus, it should be noted how many infants from one birth the individual women will have brought forth, & the number of all the infants would be divided by the number of women, the number which would come forth in the quotient, surely it would stand apart from the number <of births> consisting of three <children> by a greater distance, than from <those consisting of> one; on the contrary if it must be said that the matter is, the number of chances, in which more than one is born, compared to the number of chances, in which only one is brought forth, it has a small ratio, so that safely it would be able to be neglected, if indeed someone wishes to carry back even now, indeed that number of chances, in which three or more are born, are small & able to be neglected, but by no means rarely to happen, so that twins would be produced in one birth, and from this cause it must be presumed, that two at least are about to be produced: we will reply, if not frequently, at least not more rarely it happens also, that pregnant women would cause miscarriage or that they would produce stillborns, as much as they would bring forth twins, therefore when a miscarriage or one who has died has been brought forth stillborn this must be counted as none having been born, excess will be balanced against defect, if we should establish that one is about to be born, for if among one thousand, e.g. pregnant women there should be one by chance, who would bring forth two, also there will be one in the least, who brings forth a miscarriage or stillborn, and therefore if concerning the pregnant women it must be conjectured, how many infants that woman would be about to produce, we will have one chance that two, 998 that one, & one that none would be born, accordingly the expectation will be

$$\frac{1 \cdot 2 + 998 \cdot 1 + 1 \cdot 0}{1000} = \frac{1000}{1000} = 1;$$

we conclude therefore, that the regular arrangement of this law of the third must be heeded not at all, but rather, if always the division cannot be left undecided until the nativity of the last born, the inheritance must be divided into two parts, one of which for the son, the other ought to be awarded meanwhile to the fetus in the womb; & if later it will have happened, that indeed the one would not have been brought forth alive, the remainder will be ordered accrued to the son, but if indeed more than one will have been born, so much he will decrease
54.
for himself, as much as he receives more than proper, *arg. l. 4. si pars hered. petat.*

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appears (as says Theophrastus) *the lawgivers pass over*, that is, legislators disregard. And therefore also if she is about to bring forth one, not of the half share, but of the fourth he will be an heir meanwhile.

Chapter 9

**CONCERNING CREDIBILITY OF WITNESSES &
CONCERNING SUSPICIONS; AND ALSO
CONCERNING COMMODATION,[1] WHETHER HE
WOULD BE BOUND TO FULFILL THE CASE, IF THE
PROPERTY WOULD NOT BE DESTROYED BY
LENDING**

Since in testimonies the credibility of witnesses, before they would be admitted, must be examined carefully by means of *l. 2. & 3. ff. de testib.* not by this matter will this have been to share the Rule, with the power of which one measures the credibility of anyone, & one would be permitted to suppose, how much the probability would be, that one would speak the truth, or one would not speak; however this is the rule: Divide the number of chances, in which one will have been observed to speak truths, by the sum of those chances and the chances in which one has been observed to lie, & you will have the degree of credibility; or if several men themselves having been approved by credibility would offer a testimony of truth, others not the lesser having been approved by credibility would accuse <the testimony> itself of perfidy, divide the number of those men by the sum of both.

In like manner if someone should be under suspicion of any crime, & several circumstances or evidences should serve against him, nevertheless from which individual <evidences> it is unable to be proved, that a crime has been committed, we will be able to discover, how great the probability would be, that someone would be guilty or innocent; for if, e.g. in the individual circumstances it should be twice more probable, that someone would be innocent, than guilty, & at first no evidence were to be at hand against the accused, the innocence of him would be put forth without doubt, i.e. it would be worth 1. if indeed one evidence should be at hand, the innocence will be worth less than 1, & indeed (because there are two cases that the crime would not have been committed and one that it would have) $(2 \cdot 1 + 1 \cdot 0)/3 = 2/3$. If now supporting evidence should be added, there will be two cases that this evidence would be false, i.e. that

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[1] A commodation is the permitted use of a thing without compensation. In other words, it is a free lending of the thing.

only one evidence would remain, in which case the innocence, as in the manner we discover, is worth 2/3, & one case that it would be true, i.e. that a crime would have been committed, and indeed the innocence will be worth

$$\frac{2 \cdot \frac{2}{3} + 1 \cdot 0}{3} = \frac{4}{9};$$

Thus if three evidences were to be at hand, the innocence would be worth

$$\frac{2 \cdot \frac{4}{9} + 1 \cdot 0}{3} = \frac{8}{27};$$

if four

$$\frac{2 \cdot \frac{8}{27} + 1 \cdot 0}{3} = \frac{16}{81};$$

and thus hereafter; whence it is clear, that the innocence continues to decrease in Geometric proportion, & always is equal to the fraction 2/3 raised to this power, the index of which is equal to the number of evidences, whence so that if 10, e.g. evidences were to be against someone, the innocence of him would be worth

$$\left(\frac{2}{3}\right)^{10} = \frac{1024}{59049}$$

which is so small, so that as a rule it would be morally certain, that a crime has been committed.

Questions various in kind still would remain to be discussed, but lest I would be more long-winded, than it is proper, I would attach only one question to the place of the End, concerning commodation, namely, whether there would be held the valuation for a property, or another of the same kind, if indeed it will have been destroyed by an accidental chance, which of itself was not to have averted; and very many counselors at law plainly deny this through *l. 5. § 4. l. 18. pr. ff. commod. l. 1. C. eod. l. 1. § 4. V. O. § 2. I. quib. mod. re contr. obl.*

However I would think in this matter that it must be distinguished with Pufendorf *de I. N. & G. lib. 5. cap. 4. § 6.* & Mornacio *ad l. 1. C. commod.* whether the property had been destroyed altogether by the owner, if either more than something <the valuation> should not be given by the property having been lent, or in truth less; in the first case the commodation is not held, but not so in the latter, for it is unjust, that one's own service of commodation is harmful *l. 7. ff. testam. quemadm. aper.* which could happen, if he himself ought to endure the chance, & likewise beyond free use he ought to make no use of his own property also.

Indeed some men will rule, that it is unjust, that someone would suffer for the sake of meager gain and momentary great loss, indeed it is just as if he would place 100. Imperials against one; indeed we respond to this from our Art of Conjecture, that this itself on the contrary is not unfair, for in games, as above we warned in *Chapter 7*, no inequality exists, if the danger of losing and the expectation of gain should be in proportion to the things on both sides,

in respect to which there is contention; but plainly there is, that out of one hundred, no indeed, one thousand or more events hardly once such an accidental case happens, whence just as it is not unfair, that someone would lay down one hundred Imperials against one, if in return he should have an expectation of winning one hundred times more than of losing: therefore this ought not to be counted as unjust, so that the commodation for use of the property, which itself is given up without recompense, a loss, if it will have happened by accidental chance, he would be compelled to restore by commodation.

56.

Therefore concerning the Use of the Art of Conjecture in Law for the present time it is sufficient to have expressed these <ideas>.

COROLLARIES

1. *Honest acts are not brought about on account of themselves and disgraceful <acts are not brought about> in advance according to the law.*

2. *It is permitted to demand the interest of interest, this can be demonstrated Mathematically.*

3. *The Lex Falcidia also has a place on the occasion of death for the taking of properties.*

4. *In the case of law 13. ff. de lib. & posth. the inheritance ought to be divided into 18. parts, of which 5 to the wife, 10. to the son & 3 to the daughter must be adjudged.*

5. *Among the Roman months February never had 29. days see* l. 98. § 2. de V. S.

6. *Counselors at Law are able to argue over nothing,* arg. l. 70. § 9. de legat. 3.

To the Most Learned Master, the Candidate.

Cum tot sint casus, quibus est Bernoullia proles
Nominis æternum semper adepta decus;
Cur non hoc de te quoque conjectare liceret,
Ars Conjectandi cum mihi monstret idem? [2]

p.
F. C. J. St.

END[3]

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[2] *Since so many cases exist, in which the Bernoullian descendant*
has obtained always eternal glory of name;
Why should one not be permitted to conjecture this concerning you also,
Would the Art of Conjecturing point out the same thing to me?

[3]Translation of this work by Richard J. Pulskamp, Department of Mathematics and Computer Science, Xavier University, Cincinnati, OH and A. Berra with the assistance of others as noted.