III. DE FRACTIONIBUS ALGEBRAICIS RADICALITATE IMMUNIBUS AD FRACTIONES SIMPLICIORES REDUCENDIS, DEQUE SUMMANDIS TERMINIS QUARUMDAM SERIERUM ÆQUALI INTERVALLO A SE DISTANTIBUS.

AUCTORE *ABRAHAMO DE MOIVRE*, S.R. SOCIO.

To the most erudite Man *JOHN MACHIN*, to the Secretary of the Royal Society, *A. de Moivre*, S.P.

I send to you a certain excerpt from my Writings presented before the Royal Society 5 May 1720, which on the same day they have placed near the hand of the Secretary. One part of these Writings now through a period of two years had been deposited with the Celebrated President; moreover it preserved the Demonstrations of certain Propositions in the Book published in English by me which has been entitled THE DOCTRINE OF CHANCES. *The other part preserved a more copious explanation of the Demonstrations which it had touched more lightly previously. Now when frequently you have urged me that I should compose certain selected Propositions out of these Writings chosen of universal law, seeing that judging in the case of those to be discovered certain ones may be able to be applied to things of greater moment than the speculation of games may be; to this your desire finally I obey, and therefore more willingly, by which I myself seem by any right likewise by You to be able to obtain by request that you must not resist a longer time to bring forward to the Public the most beautiful inventions of yours.* Farewell.

2 August
1722.

PROPOSITION I.

If there is any Fraction whatsoever

$$\frac{1}{1 - ex + fxx - gx^3 \text{ \&c.}},$$

of which the Numerator is a given Quantity, & the Denominator is any multinomial composed out of givens, $1,\ e,\ f,\ g,$ *&c. in indeterminate* x*, I say the aforesaid Fraction to be reducible to simpler Fractions.*

First Case.

Let the proposed Fraction be

$$\frac{1}{1 - ex + fxx};$$

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Translated by Richard J. Pulskamp, Department of Mathematics and Computer Science, Xavier University, Cincinnati, OH. .

make the Denominator $1 - ex + fxx = 0$, and let $\frac{1}{m}$, $\frac{1}{p}$ be the roots of this Equation, or with $xx - ex + f = 0$ made, let $m,\ p$ be the roots of the new Equation, and make

$$A = \frac{m}{m-p},$$

and

$$B = \frac{p}{p-m},$$

& the proposed Fraction will be the equal of the sum

$$\frac{A}{1-mx} + \frac{B}{1-px}.$$

Second Case.

Let the proposed Fraction be

$$\frac{1}{1-ex+ff-gx^3};$$

there may be made $x^3 - exx + fx - g = 0$, and let $m,\ p,\ q,$ be the roots of this Equation, put

$$A = \frac{mm}{\overline{m-p}\times\overline{m-q}},$$

$$B = \frac{pp}{\overline{p-m}\times\overline{p-q}},$$

$$C = \frac{qq}{\overline{q-m}\times\overline{q-p}},$$

& the proposed Fraction will be the equal of the sum

$$\frac{A}{1-mx} + \frac{B}{1-px} + \frac{C}{1-qx}.$$

Third Case.

Let the proposed Fraction be

$$\frac{1}{1-ex+fxx-gx^3+hx^4}.$$

There may be made $x^4 - ex^3 + fxx - gx + h = 0$, and let $m,\ p,\ q,\ s,$ be the roots of this Equation, put

$$A = \frac{m^3}{\overline{m-p}\times\overline{m-q}\times\overline{m-s}},$$

$$B = \frac{p^3}{\overline{p-m}\times\overline{p-q}\times\overline{p-s}},$$

$$C = \frac{q^3}{\overline{q-m}\times\overline{q-p}\times\overline{q-s}},$$

$$D = \frac{s^3}{\overline{s-m}\times\overline{s-p}\times\overline{s-q}},$$

and the proposed Fraction will be the equal of the sum

$$\frac{A}{1-mx} + \frac{B}{1-px} + \frac{C}{1-qx} + \frac{D}{1-sx}.$$

Fourth Case.

Let the proposed Fraction be

$$\frac{1}{1-ex+fxx-gx^3+hx^4-kx^5}.$$

There may be made $x^5-ex^4+fx^3-gxx+hx-k=0$, and let m, p, q, s, t, be the Roots of this equation, put

$$A=\frac{m^4}{\overline{m-p}\times\overline{m-q}\times\overline{m-s}\times\overline{m-t}},$$
$$B=\frac{p^4}{\overline{p-m}\times\overline{p-q}\times\overline{p-s}\times\overline{p-t}},$$
$$C=\frac{q^4}{\overline{q-m}\times\overline{q-p}\times\overline{q-s}\times\overline{q-t}},$$
$$D=\frac{s^4}{\overline{s-m}\times\overline{s-p}\times\overline{s-q}\times\overline{s-t}},$$
$$E=\frac{t^4}{\overline{t-m}\times\overline{t-p}\times\overline{t-q}\times\overline{t-s}},$$

and the proposed Fraction will be the equal of the sum

$$\frac{A}{1-mx}+\frac{B}{1-px}+\frac{C}{1-qx}+\frac{D}{1-sx}+\frac{E}{1-tx}.$$

Thus the Law of Reduction produces itself with one alone to consider, and so there is the easy continuation of this that it may be useless to explain that in words.

Corollary I.

If all the Roots are equals, the proposed Fraction will not be able to be reduced to the simpler.

Corollary II.

If some Roots are equals, however the others unequal, the proposed fraction will be able to be reduced to the simpler; let for the sake of an example the proposed fraction be

$$\frac{1}{1-ex+fxx-gx^3},$$

and from the fact that it has been prescribed $x^3-exx+fx-g=0$. Let the Roots of this equation be m, p, q, of which m & p are equals: the simple fractions will be in what the proposed is resolved

$$\frac{mm}{\overline{m-p}\times\overline{m-q}\times\overline{1-mx}}+\frac{pp}{\overline{p-m}\times\overline{p-q}\times\overline{1-px}}+\frac{qq}{\overline{q-m}\times\overline{q-p}\times\overline{1-qx}};$$

the first two may add into one sum, & the sum will be (with the Numerator & Denominator divided by $m-p$)

$$\frac{mp-q\times\overline{m+p}+mpqx}{\overline{m-q}\times\overline{p-q}+\overline{1-mx}\times\overline{1-px}},$$

or

$$\frac{mm-2qm+mmqx}{\overline{m-q}^2\times\overline{1-mx}^2},$$

or

$$\frac{m}{\overline{m-q}\times\overline{1-mx}^2}-\frac{qm}{\overline{m-q}^2\times\overline{1-mx}},$$

and the reduced Fractions will be

$$\frac{m}{\overline{m-q}\times\overline{1-mx}^2}-\frac{qm}{\overline{m-q}^2\times\overline{1-mx}^2}+\frac{qq}{\overline{m-q}^2\times\overline{1-qx}}.$$

Corollary III.

If the simple Fractions into which the proposed Fraction is resolved may involve imaginary Quantities, then what there is of the imaginary will always be destroyed by the addition of two or more fractions assumed with even number.

Corollary IV.

From the combination of the simple Fractions, & proper limitation of the Roots, many Theorems will come into existence in which a certain elegance will be the least to be scorned. For example let the proposed fraction be

$$\frac{1}{1-ex+fxx-gx^3+hx^4},$$

and by the fact that previously $x^4-ex^3+fxx-gx+h=0$. Let $m,\ p,\ q,\ s,$ be the Roots of the Equation, and let the Fractions into which the proposed is resolved,

$$\frac{A}{1-mx}+\frac{B}{1-px}+\frac{C}{1-qx}+\frac{D}{1-sx}.$$

Let be put $q=-m$, and $s=-p$; and let the first two be added at the same time, and similarly the last two, & the proposed fraction will be reduced to

$$\frac{m+p-mpx}{2\times\overline{m+p}\times\overline{1-mx}\times\overline{1-px}}+\frac{m+p+mpx}{2\times\overline{m+p}\times\overline{1+mx}\times\overline{1+px}}$$

if indeed is put $p=-m$, and $s=-q$, & the first two added, and likewise the last two, the proposed Fraction will be reduced to

$$\frac{mm}{\overline{mm-qq}\times\overline{1-mmxx}}+\frac{qq}{\overline{qq-mm}\times\overline{1-qqxx}}.$$

PROPOSITION II.

If there is any fraction whatever of which the Numerator is a given Quantity, & the Denominator is a Trinomial or Quadrinomial or Quinquinomial, &c. not affected by a root & composed in whatever manner out of the givens, $1,\ e,\ f,\ g,\ h,$ *&c. & the indeterminate* x *and the Numerator is divided by the Denominator, in order that it has an Infinite Series; I say to be that, if any Terms of this series are summed at intervals being equally distant from one another, the infinite series thence resulting, is going to be summable.*

Example I.

Let the proposed Fraction be

$$\frac{1}{1-x-xx};$$

that may be reduced to the infinite series, certainly to

$$1+x+2xx+3x^3+5x^4+8x^5+13x^6+21x^7+34x^8 \text{ \&c.}$$

and all alternate terms may be summed, by beginning with the first, and likewise all alternate Terms may be summed, beginning with the second, and thus two series may be constructed,

Evidently,

$$1+2xx+5x^4+13x^6+34x^8 \text{ \&c.}$$

$$x+3x^3+8x^5+21x^7+55x^9 \text{ \&c.}$$

The Denominator of the proposed Fraction may be formed, $1 - x - xx = 0$, now since the indices of the powers of the indeterminate x, in the new series surpass themselves in turn by the common difference 2, put $xx = z$, and by the power of the two equations

$$1 - x - xx = 0, \quad \& \quad xx = z,$$

let x be exterminated;[1] and there will happen $1 - 3x - zz = 0$; now besides let x be restored, & there will be $1 - 3xx + x^4 = 0$; let this equation be divided by the first, the quotient will be $1 + x - xx$; the Terms of the quotient may be summed, by means of the Terms alternately summed in the proposed series, and hence two sums will arise, $1 - xx$, & x; these sums may be established the Numerators of the two Fractions of which the common Denominator is $1 - 3xx + x^4$, and the respective sums of the new Series will be

$$\frac{1 - xx}{1 - 3xx + x^4} \quad \& \quad \frac{x}{1 - 3xx + x^4}.$$

Example II.

If the sums of the terms being distant from one another at two intervals are desired, it happens as before $1 - x - xx = 0$, now since the indices of the powers in the new series surpass themselves by turns with a common difference 3, let be put $x^3 = z$, & it[2] becomes $1 - 4z - zz = 0$, and with x restored, there will become $1 - 4x^3 - x^6 = 0$; let $1 - 4x^3 - x^6$ be divided by $1 - x - xx$, the quotient will be $1 + x + 2xx - x^3 + x^4$, of which the terms assumed in succession at intervals two times, three times will construct the sums, one may see, $1 - x^3$, $x + x^4$, $2xx$, which assumed figures, will be of the Numerators, of the three Fractions, in which if the common Denominator $1 - 4x^3 - x^6$ is placed, there will be the three Fractions,

$$\frac{1 - x^3}{1 - 4x^3 - x^6}, \qquad \frac{x + x^4}{1 - 4x^3 - x^6}, \qquad \frac{2xx}{1 - 4x^3 - x^6}$$

the three sums of all Terms being a distance from one another by an interval of two, by starting respectively with the first, with the second & with the third Term; and by the same method it is permitted to gather together the sums of the terms being distant from one another at intervals of three times or four times or five times, or the denominator is any quadrinomial, or multinomial composed out of finite terms.

PROPOSITION III.

If Unity is divided by a Trinomial composed in whatever manner out of the givens $1, e, f, g,$ *&c. & the indeterminate x; I say any Term of the Series resulting out of this division, to be assignable.*

Let the Trinomial be $1 - ex + fxx$ make $xx - ex + f = 0$, let m & p be the roots of the Equation; let $l + 1$ be the locus of the desired term, this is, l expresses the interval between the first Term & the sought Term, make

$$A = \frac{m}{m - p} \qquad B = \frac{p}{p - m}.$$

And the desired Term will be $\overline{Am^l + Bp^l} \times x^l$.

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[1] *Translator's note*: If we solve the first equation for x by means of the quadratic formula, then $x = \frac{-1 \pm \sqrt{5}}{2} = \sqrt{z}$. Therefore $z = \left(\frac{3 + \sqrt{5}}{2}\right)^2$ or $(2z - 3)^2 = 5$. We find that $1 - 3x^2 + z^4 = (1 - x - x^2)(1 + x - x^2)$.

[2] *Translator's note*: Moivre does not show how he derives the polynomial in z. We are convinced it must be in the same manner as shown in the previous note. Putting $z = \left(\frac{3 + \sqrt{5}}{2}\right)^3$ we find that $z = -2 \pm \sqrt{5}$. Hence $(z + 2)^2 = 5$. In this case we discover $1 - 4x^3 - x^6 = (1 - x - x^2)(1 + x + 2x^2 - x^3 + x^4)$.

In the same manner if Unity is divided by a quadrinomial $1 - ex + fxx - gx^3$, put $x^3 - exx + fx - g = 0$, and let $m,\ p,\ q,$ be the roots of the Equation, make

$$A = \frac{mm}{\overline{m-p} \times \overline{m-q}}, \quad B = \frac{pp}{\overline{p-m} \times \overline{p-q}}, \quad C = \frac{qq}{\overline{q-m} \times \overline{q-p}}.$$

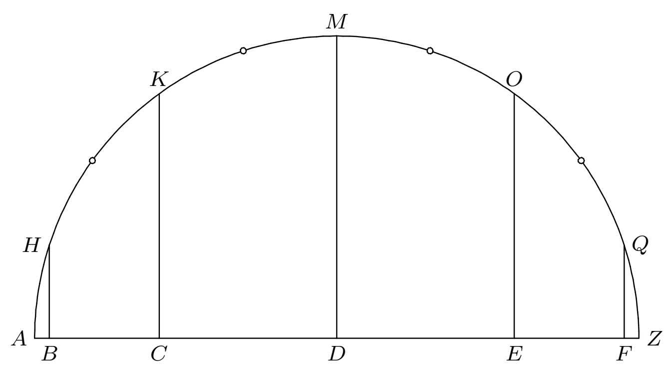
And the desired Term will be $\overline{Am^l + Bp^l + Cq^l} \times x^l$, & the same law obtains for any multinomial whatsoever.

PROBLEM

A & B of whom the Skills are in given ratio one may see as a to b, play with the condition, that as often A will have defeated one game, B must surrender to him one coin: as often indeed B will have defeated, A must surrender to him one coin: & not before they cease the game, but one of them gained all coins of the other; there is sought how much probability may be that the contest must finish within a given number of games x, or ceasing by that number.

First Case.

Let n be the number of coins which both of the Gamesters may have; furthermore let n be an even number, and let be put a to b to have the ratio of equality.



With Center D, with Interval $DA = 1$, let the semicircumference AMZ be described which is divided into as many equal parts as there are units in n, then from the first H, the third K, the fifth M &c. each odd term of the division, let $HB,\ KC,\ MD,\ OE,\ QF$ &c. be dropped perpendicular to the diameter. Put

$$Q = \frac{HB^{x+1}}{AB^{\frac{1}{2}x+1}} - \frac{CK^{x+1}}{AC^{\frac{1}{2}x+1}} + \frac{DM^{x+1}}{AD^{\frac{1}{2}x+1}} - \frac{EO^{x+1}}{AE^{\frac{1}{2}x+1}} + \frac{QF^{x+1}}{AF^{\frac{1}{2}x+1}} \text{ \&c.}$$

until all sines are exhausted: by which fact, the probability of the contest being finished within no more games than x, to the probability of not being finished, will be as $2^{\frac{1}{2}x+1}n - Q$ to Q, exactly.[3]

Corollary I.

[3]*Translator's note*: Since this is the upper half of the unit circle, $HB = \sin(\pi/n)$ and $AB =$versin(π/n). In modern notation, we have

$$Q = \sum_{k=1}^{n/2} \frac{\sin\left(\frac{(2k-1)\pi}{n}\right)^{x+1}}{\left(1 - \cos\left(\frac{(2k-1)\pi}{n}\right)\right)^{x/2+1}}.$$

If the first Term $\frac{HB^{x+1}}{AB^{\frac{1}{2}x+1}}$ is assumed for Q by neglecting the others, the approximation will be had sufficient unless by chance the number x is very small.

Example.

Let n be the number of coins which each of the Gamesters may have $= 10$. Furthermore let $x = 76$. If the first term may be assume for Q & the others may be neglected, the probability of the contest being finished within no more than 76 games to the probability of not being finished will be discovered as 50747 to 49235, if indeed it is assumed for Q the first two terms by neglecting the others, the ratio of the probabilities will be discovered as 50743 to 49247.

Corollary II.

To discover in how many games, the probabilities of the contest being finished & not being finished will be equals.

Solution.

Let the one Term $\frac{HB^{x+1}}{AB^{\frac{1}{2}x+1}}$ be put for Q, and let be made $2^{\frac{1}{2}x-1}n - Q = Q$. And by putting n the largest number it will be discovered,[4] $x = 0.756nn$ approximately, by considerably more than $\frac{3}{4}nn$.

Second Case.

Let n be an odd number, and let a to b be put to have the ratio of equality.

[4]*Translator's note*: It is certainly not obvious how this constant is derived. Indeed it is $\frac{8}{\pi^2}\ln(8/\pi)$. To see this we may proceed as follows. By the conclusion of the first case,

$$p = \Pr(\text{the game ceases in more than } x \text{ games}) = \frac{\sin(\pi/n)^{x+1}}{n2^{x/2-1}(1-\cos(\pi/n))^{x/2+1}}$$

which may be rewritten as

$$p = \frac{2\sin(\pi/n)}{n(1-\cos(\pi/n))} \cdot \left(\frac{\sin(\pi/n)}{\sqrt{2(1-\cos(\pi/n)}}\right)^x$$

Since we may assume n large, $\sin(\pi/n) \approx \pi/n$ and $\cos(\pi/n) \approx 1 + \pi^2/2n^2$. Similarly, if y is small,

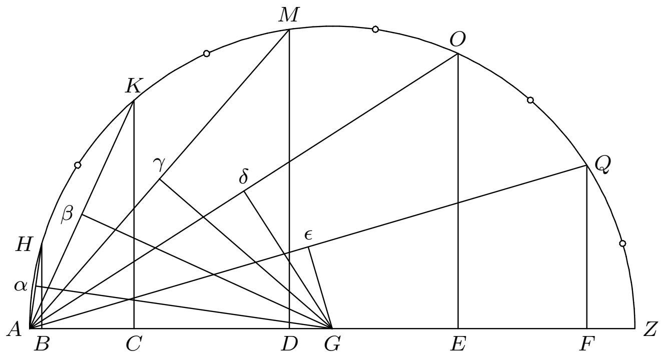
$$\frac{\sin(y)}{\sqrt{2(1-\cos(y)}} \approx 1 - \frac{y^2}{8}.$$

Hence

$$p \approx \frac{4}{\pi}\left(1 - \frac{\pi^2}{8n^2}\right)^x$$

Finally, putting $p = 1/2$ and solving for x, we find

$$x = \frac{8\ln(8/\pi)}{\pi^2}n^2 \approx .7576n^2.$$



With Center G, interval GA let the semicircumference AMZ be described which is divided into as many equal parts, as there are units in n; then from the first H, the third K, the fifth M, & each odd term of the division, let HB, KC, MD, OE, QF &c. be dropped perpendicular to the diameter; from the extremity of the diameter A namely conterminous to the first arc, let AH, AK, AM, &c be prolonged underneath; to which let $G\alpha$, $G\beta$, $G\gamma$, $G\delta$, $G\epsilon$, &c. be prolonged perpendicular from the Center G; let be put

$$Q = \frac{BH^x \times G\alpha}{AB^{\frac{x+1}{2}}} - \frac{CK^x \times G\beta}{AC^{\frac{x+1}{2}}} + \frac{DM^x \times G\gamma}{AD^{\frac{x+1}{2}}} - \frac{EO^x \times G\delta}{AE^{\frac{x+1}{2}}} + \frac{FQ^x \times G\epsilon}{AF^{\frac{x+1}{2}}} \text{ \&c.}$$

by which fact, the probabilities of the contest being finished within no more than x games, to the probability of not being finished, as $2^{\frac{x-3}{2}}n - Q$ to Q exactly.

Corollary I.

If the first term $\frac{BH^x \times G\alpha}{AB^{\frac{x+1}{2}}}$ is assumed for Q with the remaining neglected, the approximation will be considered sufficient.

Example.

Let n be the number of coins which each of the Gamesters may have $= 45$. Furthermore let $x = 1519$. Let the first term be assumed for Q with the remaining neglected, & the probability of the contest being finished within no more games than 1519 to the probability of not being finished as 49959 to 50441, which proportion is very nearest.

Corollary II.

To discover in how many games the probabilities of the contest being finished & not finished will be equals.

Solution.

Let the one Term $\frac{BH^x \times G\alpha}{AB^{\frac{x+1}{2}}}$ be put for Q, and let there be $2^{\frac{x-3}{2}}n - Q = Q$; & with the large number n put, there will be $x = 0.756nn$ nearest considerably more than $\frac{3}{4}nn$ against what the Illustrious Montmort perceived.

Third Case.

With the others put as in the first case, let the ratio a to b be of inequality (see Fig. I.) Put

$$\frac{a^n + b^n}{\overline{a+b}^n} = L, \quad \frac{\overline{a-b}^2}{\overline{a+b}^2} = d, \quad \frac{ab}{\overline{a+b}^2} = r,$$

Make[5]

$$1, 2r :: \frac{HBq}{AB}, m :: \frac{CKq}{AC}, p :: \frac{MDq}{AD}, q :: \frac{OEq}{AE}, s :: \frac{QFq}{AF}, t.$$

Put

$$Q = \frac{HB}{2rAB + d} m^{\frac{1}{2}x} - \frac{CK}{2rAC + d} p^{\frac{1}{2}x} + \frac{MD}{2rAD + d} q^{\frac{1}{2}x} \text{ \&c.}$$

by which fact the probability of the game being finished within no more games than x to the probability of not being finished will be as $nr^{\frac{1}{2}n-1} - 2LQ$ to LQ.

Corollary II.

If the first Term $\frac{HB}{2rAB+d} m^{\frac{1}{2}x}$ is assumed for Q with the others neglected, the approximation will be held sufficient.

Fourth Case.

With the others put as in the second case, let the ratio a to b be of inequality (see Fig. 2) Put the quantities L, d, r, m, p, q, s, t, &c. be as in the third case.

Put

$$Q = \frac{BH \times G\alpha}{2rAB + d} m^{\frac{x-1}{2}} - \frac{CK \times G\beta}{2rAC + d} p^{\frac{x-1}{2}} + \frac{DM \times G\gamma}{2rAD + d} q^{\frac{x-1}{2}} \text{ \&c.}$$

by which fact the probability of the game being finished within no more than x games to the probability of not being finished will be as $nr^{\frac{n-3}{2}} - 4LQ$ to $4LQ$.

Corollary.

If the one Term $\frac{BH \times G\alpha}{2rAB+d} m^{\frac{x-1}{2}}$ is assumed for Q with the others neglected the approximation will be held sufficient.

Just as in the Geometric Progression, any Term has given ratio to the nearest preceding, so are other Progressions which are able to be established in such a way that with the first two Terms assumed at pleasure, any Term subsequent to the two nearest preceding has given relation, the Series is exposed of this sort,

$$\begin{matrix} A & B & C & D & E & F \\ 1 & +3x & +7xx & +17x^3 & +41x^4 & +99x^5 \end{matrix}, \text{ \&c. in which}$$

$C = 2Bx + 1Axx$

$D = 2Cx + 1Bxx$

$E = 2Dx + 1Cxx$

$F = 2Ex + 1Dxx$ &c.

Moreover it is permitted to call the Numerical Magnitudes $2 + 1$ *likewise assumed and joined under individual signs the Index of the Relation.*

In the same manner other series are able to be established in which the first three Terms assumed at pleasure, any Term subsequent to the preceding nearest three has given relation; the Series is exposed of this sort.

$$\begin{matrix} A & B & C & D & E & F \\ 1 & +2x & +3xx & +10x^3 & +34x^4 & +97x^5 \end{matrix}, \text{ \&c. in which}$$

$D = 3Cx - 2Bxx + 5Ax^3$

$E = 3Dx - 2Cxx + 5Bx^3$

$F = 3Ex - 2Dxx + 5Cx^3$

Moreover the Numerical Quantities $3 - 2 + 5$ *likewise assumed and joined under individual signs, compose the Index of the Relation.*

[5] *Translator's note*: The notation $a, b :: c, d$ represent $a/b = c/d$.

There are other Series in which the Relation happens according to four, or five, or six preceding Terms, &c.

But it will be permitted to call all Series of this kind recurrent according to the Relation of the Terms recurring without interruption.

PROBLEM II.

In recurrent series, out of the first two given Terms, if the relation is made according to the two preceding; or out of the first three given Terms, if the relation is made according to the three preceding, &c. and also by the given index of the relation, to discover the sum of the Terms however many it pleases of which the number is given.

The Problem is solved in the our Treatise which is entitled *The Doctrine of Chances.*

PROBLEM III.

With however many recurrent series assumed at pleasure; and with Terms, at the same intervals distant from the beginning of the series, in accordance with themselves multiplied in turn, to discover the sum of the series resulting from this multiplication.

INVESTIGATION.

I$^\circ$ Let be proposed two series, and let $m+n$ be the Index of the Relation of the first series, and $p+q$ the Index of the Relation in the second, out of the first Index $m+n$, let the Equation $xx-mx-n=0$ be formed, out of the second Index $p+q$, let the Equation $yy-py-q=0$ be formed, put $xy=z$. And by the power of these three Equations, let x & y be expunged, & the Equation

$$z^4-mpz^3-(mmq+npp+2nq)zz-mnpqz+nnqq=0$$

in which with the first term z^4 deleted, with all signs changed, and with $z=1$ put, the Index of the Relation will be obtained, by which obtained, the resulting series will be summed easily; II$^\circ$ in the same manner it is permitted to proceed, if three or four &c. recurrent series are given.

While the pages above may be appended to the previously seen, I have encountered by chance in the Acta Leipzig *of the years* 1702 & 1703*, by which I have discovered the Celebrated* Leibniz *to have made uses by nearly the same method before me what I use here in reducing Algebraic Fractions to simpler, because I must wish a warning that I may avert or minimize suspicion of myself, to have wanted to claim another man's property to myself; but the proposition which I have exhibited fairly and our third Proposition, both are deduced just as the Corollary out of the other most general Proposition which we have exhibited before to the Royal Society,* May 5 1720*; thus the Proposition keeps itself.*

Given for any recurrent series of which the first Terms may be assumed at pleasure; furthermore with the Index of Relation of the following Terms to the preceding given, to discover whatever assigned Term in this series, before the sum of the series is known.

But who will have duly examined the Solution of the Problem brought here, he certainly will perceive the former to originate from our general Proposition, of which the demonstration likewise of the methods of investigation I hope shortly I may make of common justice.