# Extract from the Mémoire sur les suites récurro-récurrentes et sur leurs usages dans la théorie des hasards.

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6. I pass now to the following Problem, which had been proposed to me on the occasion of a wager made on the lottery of the military school.

PROBLEM IV. — *A lottery being composed of a number $n$ of tickets $1, 2, 3, \ldots, n$, of which there is extracted a number $p$ at each drawing, we ask the probability that after $x$ drawings all the tickets will be extracted.*

We suppose that $S$ wagers that all the tickets will not be extracted after this number of drawings, and we seek all the cases favorable to $S$; it is clear that their number is equal:

1° To the number of cases according to which the ticket 1 is not able to be extracted after the drawing $x$;

2° To the number of cases according to which the ticket 2 is not able to be extracted, the ticket 1 being extracted;

3° To the number of cases according to which the ticket 3 is not able to be extracted, the tickets 1 and 2 being extracted, and thus in sequence; if therefore we name ${}_q y_x$ the sum of all these cases to the ticket $q$, we will have

$${}_q y_n = {}_{q-1} y_n - {}_{q-1} y_{n-1} + \left[\frac{(n-1)\cdots(n-p)}{1.2\ldots p}\right]^x,$$

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an equation which corresponds to Problem I, $q$ and $n$ being supposed variables and $x$ constant; here is how we can integrate in this particular case; putting $q$ successively equal to $1,\ 2,\ 3,\ \ldots$, we will have

$$
\begin{aligned}
{}_1y_n &= \left[\frac{(n-1)\cdots(n-p)}{1.2\ldots p}\right]^x,\\
{}_2y_n &= 2\left[\frac{(n-1)\cdots(n-p)}{1.2\ldots p}\right]^x - \left[\frac{(n-2)\cdots(n-p-1)}{1.2\ldots p}\right]^x,\\
{}_3y_n &= 3\left[\frac{(n-1)\cdots(n-p)}{1.2\ldots p}\right]^x - 3\left[\frac{(n-2)\cdots(n-p-1)}{1.2\ldots p}\right]^x + \left[\frac{(n-3)\cdots(n-p-2)}{1.2\ldots p}\right]^x,
\end{aligned}
$$

whence we will conclude easily

$$
\begin{aligned}
{}_ny_n =&n\left[\frac{(n-1)\cdots(n-p)}{1.2\ldots p}\right]^x - \frac{n(n-1)}{1.2}\left[\frac{(n-2)\cdots(n-p-1)}{1.2\ldots p}\right]^x\\
&+ \frac{n(n-1)(n-2)}{1.2.3}\left[\frac{(n-3)\cdots(n-p-2)}{1.2\ldots p}\right]^x + \cdots
\end{aligned}
$$

Now here the sum of all the possible cases is $\left[\frac{n(n-1)\cdots(n-p+1)}{1.2\ldots p}\right]^x$; naming therefore $z_x$ the probability of $S$, we will have

$$
z_x = n\left[\frac{(n-1)(n-2)\cdots(n-p)}{n(n-1)\ldots(n-p+1)}\right]^x - \frac{n(n-1)}{1.2}\left[\frac{(n-2)\cdots(n-p-1)}{n\ldots(n-p+1)}\right]^x + \cdots
$$

If we wish to apply this formula to the lottery of the military school, it is necessary, according to the nature of this lottery, to suppose $n = 90$ and $p = 5$.