EXAMEN
d'une espece de Superstition ramenée au calcul des probabilités[*]

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I suppose that in the states of Europe where legal astrology no longer fills the almanacs with predictions on the weather & on some other events, one does not know that there is still a place where the almanacs are all filled out with it. This is actually the case where Germany is found; not because the enlightened persons do not scorn these astrological dreams, but because the bulk of the nation is quite avid for them. An almanac which would be purged would lose all its influence, & if one would omit all these predictions at once, there would result a general murmur & possibly even a revolt. At least all the makers of the almanacs would be obliged to hide themselves from the eyes of the public for some time.

In what I just said there is no exaggeration. Perhaps even I have not said everything. It is much more astonishing to see the little progress that learning makes with a nation, & even a nation which cultivates itself & which studies its self-interests.

It is a phenomenon which seems rather naturally to merit some examination. One would say only among those who consult the predictions of the almanac, there are none of them who do it by simple stupidity. A good portion of them know very well that their almanac deceives them rather often. But this does not suffice to make them renounce it. They will say, for example, that the barometer misleads quite often also, and that notwithstanding this one consults it. They will add that one would not know how to require of the astrologers that they know everything, & that it is already good that they are not always mistaken. They will recall very well some cases where the almanac had predicted the weather, one would not know better. In a word each accomplishment of the predictions of the almanac give them a new authority, & makes one forgive with great pleasure to the makers of the almanacs all their contempt.

Here is nearly what one has said to me, & even rather often, so strongly in favor of the astrological predictions of the almanacs. I have in vain advanced some general reasons. I have in vain said that the makers of the almanacs set the weather by total chance, by observing simply to conform it to that which each season of the year requires; that many among them do only to copy some old almanac or to entrust the attention of it

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to the Printer: one does not believe me, & the general reasons do not make impression. One single prediction, verified by the event, makes more effect than all that. To what is joined further as how the weather forms is always interpreted in favor of the almanac, if one or the other in the least may be equivocal. If the sky is simply clouded, the almanac will guess rightly if either it sets good weather or rain or clouded. For the cloudy weather is uncertain & can incline to the good as well as to rain. If the almanac predicts variable weather, some clouds or some clouds intermingled with clearness will suffice to view as variable the best or the worst weather in the world. Finally, as the almanac assigns the weather only for the entire day, it is quite natural that one accords it some latitude in this regard. One excuses even the maker of almanacs when he is mistaken only by one or by two days. One consoles oneself in that the art of predicting the weather has not yet been advanced as far as that of predicting eclipses.

It seems to me now moreover that it is not necessary to pronounce sentence of condemnation against all those who would wish to banish from almanacs predictions on the weather. I come now to examine the same subject in a more general fashion & relatively to the principles of probabilities. The public is satisfied with the simple comparison of the prediction with the event. We suppose that it makes this comparison with more rigor than it does not; the question is to know, if by means of this comparison & setting aside all general consideration, can it correct itself of its error?

In this regard there are three different cases, which present themselves naturally enough, & which seem to offer as much as particular rules. 1. We suppose that the almanac deceives more often than it guesses rightly. In this case one must be lead to believe that its predictions are false. 2. If it deceives as often as it guesses correctly, the question will remain undecided. 3. If it guesses rightly more often that it deceives, one will continue to consult it.

These three rules seem quite right, & we have only to change subjects in order to find application of them by some enlightened persons. For in all the cases which are not able to be decided with an entire certitude, one tries to adhere to what happens, if not always, at least most often. There is nothing there which does not conform to the rules of prudence. I believe to have seen some collaboration & especially in the Philosophical Transactions, when the Marquis Poleni has used it thus to report on the variations of the barometer. He knows by experience that the weather does not always comply. That induced him to see if the exceptions were more frequent than the agreement. He found that in Padua it rained around twice more often when the barometer fell than when it climbed. Thus in the last case rain is twice less probable than in the first.

With all that the three rules that I just reported, admit some restrictions when the question is, not of the events, but of their causes. If there are many causes which compete to produce & to prevent some event, it is quite probable that in regard to certain events the causes which prevent them are stronger and more frequent than those which produce them. If therefore in these cases one compares the event simply with one of the causes which can produce it when nothing prevents it, one will find that these causes are only rarely following from the event. One will be likewise led to exclude them from the number of causes, at least if one does not recognize them for such for other reasons & independently of this examination. It is the same when an event due to some unknown causes returns more often, & when on the other hand there is some cause that one encounters likewise more often, although it is not the cause of the event. In this case one will observe rarely the cause without observing at the same time the event, & if one suspects at least some connection one will make the paralogism of non caussae ut caussae. For all that which can be deduced from these sorts of observations, it is that the event as well as the cause is observed very frequently, or else both are very ordinary.

We return to the almanacs & to the manner by which the public judges them. Whether the maker of almanacs follows the rules of legal astrology, or if he marks the weather following his fantasy, or finally if he decides by lot what days he must set good or bad weather; all this comes back nearly to the same. The weather as it occurs effectively follows none of these rules, because it is produced by an infinity of causes & varies nearly as fortune & misfortune in the games of chance, because again in these games the determinant causes vary in an infinity of ways.

We suppose now that the maker of almanacs nuance & vary his predictions so that he assigns 30 different modifications of weather. We give further to the weather that it makes effectively 30 different modifications. In this case the public would be quite deceived, because the almanac would agree only very rarely with the weather as it is. The calculation that one can make on that subject corresponds to the one of the *game of rencontre*, which M. *Euler* has written in the Memoirs of the Academy for the year 1751. But as he has calculated only the case where any one encounter wins the game, I will give a general solution of it. Let there be therefore n things, & let each have its assigned place. The question is, in how many ways can they be transposed so that there be none of them, or there be 1 of them or 2 or 3 or 4 or 5 &c. in their place. First one knows that the number of possible cases is $= 1.2.3.4.5 \cdots n$. Next it is clear that among these cases there is only one where each piece is in its place. Moreover, if there are displacements of them, there would be at least two of them. For in order to displace one, it is necessary to displace another of them. Now these two pieces can be taken in $n \cdot \frac{n-1}{2}$ different ways. But whatever it be that one takes, there is only one way to displace them, this is to put each in the place of the other. Thus the number of cases for 2 displaced pieces is $= n \cdot \frac{n-1}{2}$. We pass to 3 pieces. They can first be chosen in $n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}$ ways. But there are only two ways to displace them. For one must be taken in the place of one of the two others, & the one in the place of which one puts it, must be taken in the place of the third. Thus the number of possible cases for 3 displaced pieces is $= n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}$. By continuing in this fashion one will find the possible cases for $0,\ 2,\ 3,\ 4,\ 5$, &c. displaced pieces, expressed by the terms of the sequence

$$x = 1 + \left(n \cdot \frac{n-1}{2}\right) + 2 \cdot \left(n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}\right) + 9 \cdot \left(n \cdots \frac{n-3}{4}\right)$$
$$+ 44 \left(n \cdots \frac{n-4}{5}\right) + 265 \left(n \cdots \frac{n-5}{6}\right) + \text{\&c.}$$

of which the sum is equal to the product $1.2.3.4.5 \cdots n$.

This sequence is composed of the terms of the binomial formula of Newton multiplied by the coefficients $1,\ 2,\ 9,\ 44,\ 265,\ 1854,\ 14833$ &c. and these coefficients are such that

$$\begin{array}{rcr} 2 &=& 1.3-1, \\ 9 &=& 2.4+1, \\ 44 &=& 9.5-1, \\ 265 &=& 44.6+1, \\ 1854 &=& 265.7-1, \\ 14833 &=& 1854.8+1, \\ \text{\&c.} && \end{array}$$

One can again find these numbers with the help of the sequence

$$x = 1 + \left(n\cdot\frac{n-1}{2}\right) + 2\cdot\left(n\cdot\frac{n-1}{2}\cdot\frac{n-2}{3}\right) + 9\cdot\left(n\cdots\frac{n-3}{4}\right) + 44\left(n\cdots\frac{n-4}{5}\right) + 265\left(n\cdots\frac{n-5}{6}\right) + \text{\&c.}$$

by placing successively for n the values $1,\ 2,\ 3,\ 4$ &c. For the sum x being $= 1.2.3\cdots n$, it is given independently of the sequence. This requires that one of the coefficients $1,\ 2,\ 9,\ 44$ &c. can be regarded as unknown. We suppose

$$x = a + b\left(n\cdot\frac{n-1}{2}\right) + c\left(n\cdot\frac{n-1}{2}\cdot\frac{n-2}{3}\right) + d\left(n\cdots\frac{n-3}{4}\right) + \text{\&c.}$$

by making

$$\begin{array}{rlll} n=1, & \text{one will have} & x=1 & =a, \\ 2, & & x=1.2 & =a+b, \\ 3, & & x=1.2.3 & =a+3b+c, \\ 4, & & x=1.2.3.4 & =a+6b+4c+d, \\ & & \text{\&c.} & \end{array}$$

this which gives

$$\begin{array}{l} a=1, \\ b=1, \\ c=2, \\ d=9, \\ e=44, \\ \quad\text{\&c.} \end{array}$$

& consequently

$$x = 1 + \left(n\cdot\frac{n-1}{2}\right) + 2\left(n\cdot\frac{n-1}{2}\cdot\frac{n-2}{3}\right) + 9\left(n\cdots\frac{n-3}{4}\right) + 44\left(n\cdots\frac{n-4}{5}\right) + \text{\&c.}$$

By means of this sequence one can form the particular sequences for each value of n, by making successively $n = 1,\ 2,\ 3,\ 4$ &c. Thus for example

n	x	Possible	cases						
1	1= 1								
2	2= 1	+1							
3	6= 1	+3	+2						
4	24= 1	+6	+8	+9					
5	120= 1	+10	+20	+45	+44				
6	720= 1	+15	+40	+135	+264	+265			
7	5040= 1	+21	+70	+315	+924	+1855	+1854		
8	40320= 1	+28	+112	+630	+2464	+7420	+14832	+14833	
9	362880= 1	+36	+168	+1134	+5544	+22260	+66744	+33497	+133496
&c.	&c.								
Pieces displaced	0,	2,	3,	4,	5,	6,	7,	8,	9&c

It is easy to see that by continuing this Table to $n = 30$, the number of possible cases will become an enormous number, & that the number of those which would be favorable to the predictions of the almanac, will be considerably smaller. For in general for x cases he must guess nx times, but he guesses only

$$\begin{aligned} y = n + (n-2)\left(n \cdot \frac{n-1}{2}\right) + 2 \cdot (n-3)\left(n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}\right) \\ + 9(n-4)\left(n \cdots \frac{n-3}{4}\right) \\ + 44(n-5)\left(n \cdots \frac{n-4}{5}\right) \\ + \text{\&c.} \end{aligned}$$

times. This series is found equal to that which we have given for x. For one has

$$\begin{aligned} \frac{y}{n} = 1 + (n-1) \cdot \frac{n-2}{2} + 2 \cdot (n-1)\frac{n-2}{2} \cdot \frac{n-3}{3} \\ + 9 \cdot (n-1)\frac{n-2}{2} \cdot \frac{n-3}{3} \cdot \frac{n-4}{4} \\ + 44(n-1)\frac{n-2}{2} \cdot \frac{n-3}{3} \cdot \frac{n-4}{4} \cdot \frac{n-5}{5} \\ + \text{\&c.} \end{aligned}$$

This is the value of x for the number of pieces $(n-1)$. And thus one will have

$$\frac{y}{n} = 1.2.3.4 \cdots (n-1).$$

Therefore

$$y = 1.2.3.4 \cdots n = x.$$

As therefore in the case of n different modifications the almanac, instead of guessing nx times, guess only $y = x$ times, it is clear that he deceives $n-1$ times more than he not deceive, so that by supposing $n = 30$, it follows that among 30 predictions there will be only one which is realized.

But the makers of almanacs refrain well to give to the weather as it occurs so many different modifications, although they do not fail to vary the weather, by employing some synonyms & other equivalent expressions. Thus the public is rather well disposed to not take these expressions strictly. All reduce themselves therefore to two general modifications, this is good weather & bad weather, or in all cases to three, & then it is clear weather, cloudy weather & rain.

Now we have just seen that in the case of 2 modifications one has $n = 2$, & this makes that the half of the predictions of the almanac are verified by the event. It can even be that this is beyond the half. There is necessary for this only one single precaution, of which the makers of almanacs could avail themselves, if they do not put it in effect; this is to predict the weather tranquil 2 or 3 times less often than cloudy weather, with the exception of March & September where the good weather seem to predominate.

We suppose that in reality the number of good days is $= a$, that of the cloudy days $= b$. If therefore the maker of almanacs distributes these days in his almanac in any fashion, the number of all the possible cases will be expressed by the square of $(a + b) = a^2 + 2ab + b^2$. Among these cases there will be

aa, where good weather agrees with the almanac,
ab, where good weather contradicts the almanac,
ab, where cloudy weather contradicts the almanac,
bb, where cloudy weather agrees with the almanac.

Thus the favorable cases in the almanac are $= aa + bb$, & the cases where they refute it, are $= 2ab$. But one knows that always $aa + bb > 2ab$. Thus there will be more cases favorable to the almanac than there are of contraries to it.

This rule will necessarily hold as soon as the weather is predicted in the almanac in the same ratio as it has actually. But if the maker of almanacs does not know this ratio for lack of consulting the meteorological observations, there is only to distribute the good weather & the cloudy weather in equal proportion, & thence he gets to see half of his predictions confirmed by the event.

Let the number of good days $= a$, those of cloudy days $= b$. Let one put $a+b = 2c$. One will set in the almanac c cloudy days & as many good days, by intermingling them in any fashion. The number of possible cases is again the square of $a + b = 2c(a + b)$. Among these cases there will be

ac, where good weather agrees with the almanac,
ac, where it does not agree,
bc, where cloudy weather does not agree,
bc, where cloudy weather agrees.

Thus the favorable cases are $= ac + bc$,
and the non favorable cases $= ac + bc$.
Therefore the almanac guesses rightly as many times as it does not guess rightly. Here therefore the balance is entirely equal at least. In the preceding calculation it leans even in favor of the almanac.

We recall now that the vulgar judges only after the comparison that he makes of the almanac with the weather, & we will infer from it that if he continues to consult the almanac it is not by a stupid superstition, but because he finds in it the predictions at the very least as many times verified as refuted by the event, & that in this regard he judges following the same rules of prudence by which all enlightened men serve themselves, when in order to examine the goodness of a rule & the degree of trust that it merits, there is no other means than to compare the number of cases where it takes place, with the number of cases where it is in default. For when even this comparison would turn to the advantage of the rule, it will not follow on account of this that the rule is based on the nature of the cases to which one wishes to apply it. It can to them be even stranger if the astrological rules are not in the changes of the air.