CORRESPONDENCE OF HUYGENS REGARDING GAMES OF CHANCE

EXTRACTED FROM VOLUMES I AND II
OF THE
OEUVRES COMPLÈTES
OF
CHRISTIAAN HUYGENS

No. 281[1]

Christiaan Huygens to [G. P. Roberval]
18 April 1656

At The Hague the 18 April 1656

SIR,

. . .I have since some days written the foundations of the calculus of the games of chance at the request of Mister Schooten who wishes to print it, where I have among others proposed one such question. When I play against another with two dice, on the condition that I will win as soon as I will make 7 points, and that he will win it as soon as he makes 6 points; and that I give to him the dice, I demand who of the two has the advantage in this, and what. I desire much to see if you will find the same solution to this as me. I am in all my heart.

No. 291[2]

P. de Carcavi to Christiaan Huygens
20 May 1656
Chr. Huygens responds in No. 297.

From Paris this 20th May 1656

SIR,

Not having the honor to be known to you, you will find perhaps strange the liberty which I take writing to you, but Mr. Mylon having wished me to procure this benefit when you were in France and having expressed to me that you had taken the trouble to come to that house,[3] I have believed, Sir that I should not only render to you all the acknowledgments and all the appreciations that I can, but still to preserve this occasion in order to show to you a part of the esteem that I make of your merit, and the first acquaintance of it having been given to me it is a long time through the late Father Mersenne, who showed me some of your good speculations, I initiated since then to honor you and to render the respects which are due to your virtue. Permit me therefore if it pleases you to render to you some

Translated by Richard J. Pulskamp, Department of Mathematics and Computer Science, Xavier University, Cincinnati, Ohio. Printed on February 1, 2009.

[1]Tome I. page 404.

[2]Tome I page 418.

[3]This is the house of Duc de Liancourt.

true testimonies and to offer to you all that which can rest with me. I know not much in mathematics but I have a great passion for that science, and as you are the most advanced in it, I would hope to procure from you some satisfaction by the intermediation of Mr. de Fermat who is my old friend, That Great Gentleman de Fermat who is certainly one of the premier men of Europe, and to show you some things from him which will merit your approval. This will be to me also a way to satisfy the inclination that I have for one so great a man by showing him at the same time that which you will have the kindness to send us, and the public will receive a great advantage of the communication of two so excellent persons who will show to posterity that our century cedes not at all to the one of Apollonius, of Menelaus and of Archimedes. I have sent to him by the last postal carrier that which you demand touching the division of stakes,[4] and I gave some days ago to Mr. Mylon two beautiful problems on the numbers to show them to you, and I will avail myself of it likewise to happen it if you agree to it, beseeching you very humbly to permit me in finishing this letter to implore you to continue your good meditations and to increase a Science which has made so great progress in our days. . . .

SIR,

Your very humble and obedient servant
DE CARCAVY

No. 296[5]

Christiaan Huygens to Claude Mylon
[1 June 1656]
Response to a letter of 13 May 1656. Mylon responds in No. 306.

To Mister MYLON

SIR,

Your letter of the 13th May[6] with that of Mister de Carcavy[7] arrived here only the 24th.

The problems of Mister de Fermat are entirely beautiful in the genre, and not easy to resolve, at least it seems such to me who am scarcely exercised in the questions of numbers, because I have always taken more pleasure in those of geometry. However I will try again if I can become master of them, and if not I hope that you do me the favor to send to me the solution that the author or some among you other scholars will give of them. I have taken great contentment seeing your reasoning on that which I have proposed on the game of dice, which has not deceived you in the first articles, namely when you said that the advantage of the one who has the primacy is to the one of the other as 6 to 5 when they must bring forth a certain point with one die only and that the advantages would be as 2 to 1 if the die had only two faces. All that is true, but it is necessary however that your argumentation not be so assured, because it undermines you to a false conclusion in the following cases. You say that if two players A and B play with two dice in order to bring forth a like number of points, as 7, the primacy of player A will be worth to him in regard to

[4]The problem of points.

[5]Tome I. pages 426-427.

[6]The first letter of Huygens to Mylon is missing, as also is the response of Mylon of 13 May. However this letter, No. 296, indicates well enough their contents. There results from it that besides the problem of Prop. XIV Huygens had posed to Mylon some questions on the advantage of the lead in the cases where the game is won by the one who first succeeded to make a determined coup.

[7]This must be letter No. 291, although it is dated 20 May; because it is quite certain that it is the first letter that de Carcavy wrote to Christiaan Huygens: also this letter has the annotation ℞24.

B, as 36 to 35. When you wished that the primacy give equal advantage, whatever number of points that one has to bring forth, because in that which follows after you serve yourself of this theorem as generally demonstrated. And however it is evident that there is greater advantage in the primacy when it is necessary to bring forth 7 points than when 12 of them are necessary, because one encounters more easily 7 than 12. When one draws to 7 points it follows that the advantage of the one who has the primacy is to the one of the other as 6 to 5, because by playing with one die and drawing to a certain point it is found to be such because this is equally difficult to me as to bring forth 7 points with two dice. But when one draws to 12 I find the advantages in the ratio of 36 to thirty[8] 35. When therefore you serve yourself of the composition of two ratios as you do in order to find the solution of my problem which will be of 6 to 5, and of 5 to 6 there will result the ratio of equality. Of which it is evident that such a composition of Ratios can not take place. I await with impatience that which Mister de Fermat will say during which you will permit me to keep the solution hidden which I can demonstrate to be true, and to which if any one would claim to contradict it, I would be able to win his money. Since if you were content to put 7 against 6, there would be great probability that you would lose. I am

SIR,

Your &c.

I pray you to take hold my letter here joined[9] to Mister de Carcavy and to make known to me if he is not steward of the house of Monsieur le Duc de Liancourt.[10]

No. 297[11]

Christiaan Huygens to P. de Carcavy

1 June 1656

Response to No. 291. De Carcavy responds in No. 300.

TO MISTER CARCAVY

1 June 1656

SIR,

. . .I will believe myself therefore very fortunate to be known by a person so rare by your means, and to participate through time in his good inventions. The two numerical problems that Mister Mylon has sent to me are of very difficult research and I would doubt almost if there would be a way to find some other such numbers other than by chance, if one assured me only Mister de Fermat has some certain rules, which I believe nevertheless to be of such type, that it may be necessary firstly to seek some number by accident which has certain properties, as in the example which one has given for the perfect and amicable numbers. If I were more versed than I am in some similar questions of the numbers perhaps I would not find the difficulty so great, but this is where I myself am given the least. Mister de Fermat who is exercised in all sorts of problems and particularly in those of the divisions of stakes will not have so much parallel difficulty to resolve the one which I have proposed touching the dice, which is not at all difficult to those who know the principles of this calculus and

[8] Huygens has forgotten to erase this word: trente.

[9] Letter No. 297.

[10] Roger du Plessis, Duc de Liancourt and of the Roche-Gryon, Peer of France, chevalier des ordres du Roi. Born 1598. Died 1 August 1674. After a riotous youth, he became pius. The relationship of he and his wife, Jeanne de Schomberg, with Port-Royal is well known in the history of the Jansenists.

[11] Tome I. page 427.

a little of algebra. You have given me great pleasure to have sent it to him and I will see with much contentment the solution which he will have given to it. . . .

SIR,

Your very humble and very obedient servant
Chr. H.

No. 298[12]
Christiaan Huygens to Fr. van Schooten
2 June 1656

. . . Concerning the reckoning of games which I have set the equal terms it is in order that you should not advance first what you may release by yours. I have not explained the last question with it, which I had sent to Mylon in France, who has answered nothing to the thing. This indeed further by Fermat he has arranged for relating through Mr. de Carcavy. Can it be otherwise I should wish to know this than concerning the name you should have known from you and can it be something worthy to remember you may see the progress by itself. . .

No. 300[13]
P. de Carcavy to Christiaan Huygens
22 June 1656
Response to No. 297. Chr. Huygens responds in No. 308.

From Paris this 22 June 1656

SIR,

I have received the letter which you have made the honor to write to me to which I have not been able to respond as soon as I had desired so far because of a small trip that I have made in one of the lands of Monsieur le Duc de Liencourt, because of the loss and affliction which occurred to us by the death of Monsieur le Maréchal de Schonberg. I believe Sir that you will agree to these reasons for my silence and that I will not be so unfortunate that you will accuse me of negligence and to have lacked the honor and the goodwill that you have the kindness to give evidence to me in your letter.

Mr. de Fermat has sent to me beyond some days ago the solution of that which you had proposed touching the division of Stakes, and you will see from the extract which I make for you from his letter that he has the general demonstration of this sort of question, and you will conclude certainly with us not only for the resolution of this problem but also for a quantity of many other very good speculations which we have seen from him as much in that which concerns the numbers as for geometry that he is one of the greatest talents of our century,. . .

SIR,

Your very humble and obedient servant
DE CARCAVY

[12]Tome I. pages 429-430.
[13]Tome I. pages 431-432.

No. 301[14]

P. de Fermat to P. de Carcavy

[June 1656]

Appendix to No. 300[15]

If A and B, playing with two dice so that if A brings forth 6 points with his two dice before B, brings forth 7 of them, the player A, wins, And if B, brings forth 7 before A, has brought forth, player B, will have won, and moreover player A, has the primacy.[16]

The advantage of A, to B, is as 30 to 31.

If player A, has the first time the primary and next player B, has also the primary the second time, and thus alternately in which case A, will push the dice the first time and then B, two times next, and then A, two times next and thus to the end.[17]

In this kind the share of Player A, is to the one of player B, as 10355 to 12276.

That if player A, plays firstly two times and player B, 3 times then player A, 2 times, and next player B 3 times, and thus to infinity that player A who begins never plays but two coups, and that player B, plays 3 of them, supposing always that A, seeks to restore 6, and B, 7.[18]

The part of A, to B, is as 72360 to 87451.

The questions diversify and the method changes in the game of cards for Example I propose.

If three players A, B, C, wager with 52 cards, which is the number of a complete deck, that the one who will have sooner a heart will win, by supposing that A, takes the 1st card B the 2nd and C the 3rd and that this same order is always kept until one has won.[19]

If two players play to prize[20] with 40 cards, the one undertakes to bring back prize in the first four cards which will be given to him and the other wagers that the first will not succeed, what is their share.

All these questions have some methods and some different rules, if one can not come to end I will explain them to you each with their demonstrations. The most subtle and most difficult, is that of the true share of the one who holds the dice in the game of chance against the others.

---

[14]Tome I. pages 433-434.

[15]Extract of the Letter of Mister de Fermat to Mister de Carcavi [Cl. Mylon]

[16]*Translator's note*: The turns of the players go in the sequence ABABAB...

[17]*Translator's note*: The turns of the players go in the sequence ABBAABBAABB...

[18]*Translator's note*: The turns of the players go in the sequence AABBBAABBB...

[19]*Translator's note*: This problem is never completely solved. The turns of the players are as ABCABC.... There being 52 cards of which 13 are hearts, player A wins if the first heart appears on the turns which have number 1, 4, 7,.... It is easy to verify that the probability that A wins is given by $13 \cdot \sum_{k=0}^{13} \frac{39!}{(39-3k)!} \cdot \frac{(51-3k)!}{52!} = \frac{1006853859}{2334608675}$. Likewise, player B wins if the first heart appears on turns with numbers 2, 5, 8, ... 38. In this case, the probability is given by $13 \cdot \sum_{k=0}^{12} \frac{39!}{(38-3k)!} \cdot \frac{(50-3k)!}{52!} = \frac{6072680801}{18676869400}$. Finally, player C wins if the first heart appears on a turn which is a multiple of 3 to 39. The probability is given by $13 \cdot \sum_{k=0}^{12} \frac{39!}{(37-3k)!} \cdot \frac{(49-3k)!}{52!} = \frac{4549357727}{18676869400}$. The probabilities A, B, C are thus approximately $0.431,\ 0.325, 0\ .244$ respectively.

[20]*Translator's note*: A prize is all four suits being represented in a set of 4 cards.

Let there be further if you wish two Players who play at piquet the first undertakes to have 3 aces in his first twelve cards, what is the part of the one here against the other who wagers that he will not have the three aces.

No. 306[21]

Cl. Mylon to Christiaan Huygens

23 June 1656

Response to No. 296. Chr. Huygens responds in No. 310.

At Paris this 23 June 1656

SIR,

I am not unhappy to have hit the nail on the head in a part of that which I sent the last time in order to respond to your question on the primacy of the die. I console myself to be deceived in the rest since it was the most difficult. I confess to you that through my method I must conclude the ratio of equality between the two players of whom the one who has the primacy draws to the 6 and the second to the 7 with two dice. But I have difficulty to understand why the composition of ratios does not compensate the different advantages of these Players. The subtraction of the ratios would be yet worse since you inform me that these advantages are in least ratio as of 7 to 6, and that Messrs. de Fermat and Pascal have found separately that it was as 30 to 31. Mister de Carcavi sent you the extract of the response that Mister de Fermat has made to him on your question, where you will find with what to exercise yourself, I believe that after this here you will do us well the favor to send to us your resolution & demonstration, in order to show it to these Sirs.

Your very humble and very obedient servant

MYLON

No. 308[22]

Christiaan Huygens to P. de Carcavy

6 July 1656

Response to No. 300. P. de Carcavy responds in No. 336.

To Mister DE CARCAVY

SIR,

6 July 1656

I will not say that I accept the excuses that it has pleased you to make me, because I admit not at all that they were owed to me. They are however so legitimate, that the one who would have right to demand of your letters should easily be content with them. I have seen by the solution that Mr. de Fermat has made of my Problem that he has the universal method to find all that which belongs to this matter, that which I desired solely to know in proposing it. The same ratio of 30 to 31 is in the treatise that I have sent to Mr. Schooten 2 months ago; in the same there is also a Theorem of which I serve myself in all these questions on the divisions of the game; and I will set it here, because otherwise I would not be able to show you that I am come to the end of the Problems that Mr. de Fermat has proposed. The calculation of some among those being so long that I have not enough patience in order to seek in it the last product; this is why in those there after having

[21]Tome I. page 438.

[22]Tome I. pages 442-446.

explained to you the said theorem, I will content myself to set the method through which one can arrive to it. The Theorem is this one here.

If the number of chances that one has in order to have b is p, and the number of chances that one has in order to have c is q: This is worth as much as if one had $\frac{bp+cq}{p+q}$. For example, if I had 2 chances in order to have $\frac{1}{3}$ of that which is set into the game and 5 chances in order to have $\frac{1}{2}$ of it, I multiply $\frac{1}{3}$ by 2; and $\frac{1}{2}$ by 5. Then I add together the products which are $\frac{2}{3}$ and $\frac{5}{2}$; the sum is $\frac{19}{6}$. Which I divide by $5+2$, this is 7; of which I have $\frac{19}{42}$. I say that there belongs to me $\frac{19}{42}$ of that which is set into the game.

The first of the questions of Mr. de Fermat is such. A and B play with 2 dice. A will win in bringing forth 6 points. B will win in bringing forth 7 points. A will push the die the first time; and then B two times in sequence and then A two times in sequence, and thus until one or the other has won. In order to make the parts I will name d that which is set into the game; And I will set x for the part of it which belongs to player A. Now it is evident that when A will have made the first coup, and B his two coups in sequence; and next A one of his two coups, without that neither one nor the other has encountered, that then A will have anew the same probability to win as he had at the beginning, and that consequently there will belong to him anew the same part of that which is put into the game, that is to say x.

Hence when A comes to make the first of his two coups in sequence he will have $\left\{\begin{matrix}\text{5 chances to have } d,\\ \text{and 31 chances to have } x,\end{matrix}\right\}$ because of 36 diverse coups which 2 dice produce, there are of them 5 of 6 points, that is to say which gives d to him, or that which is put into the game; and 31 which make him lack the 6 points, and thus gives to him x, putting him in a state to have again a coup to make before the turn of B is come. But $\left\{\begin{matrix}\text{5 chances to have } d,\\ \text{and 31 chances to have } x,\end{matrix}\right\}$ is worth as much by the preceding theorem as $\frac{5d+31x}{36}$. This is therefore the part of A when A makes the first of this two coups in sequence.

The coup before it is when B makes the last of his two coups, and because he wins by bringing forth 7 points which are encountered in 6 different ways, and that then A loses then with this coup A will have $\left\{\begin{matrix}\text{6 chances to have 0 or nothing}\\ \text{and 30 chances to have } \dfrac{5d+31x}{36}\end{matrix}\right\}$, because his turn will be come to make two coups in sequence, which chances are by the preceding Theorem are worth $\frac{150d+930x}{1296}$. This is therefore the part of A, when B makes the last of his 2 coups in sequence. When therefore B makes the first of his 2 coups, A will have $\left\{\begin{matrix}\text{6 chances to have 0}\\ \text{30 chances to have } \dfrac{150d+930x}{1296}\end{matrix}\right\}$ this which is worth $\frac{4500d+27900x}{46656}$. When therefore A makes the first coup of all, A will have $\left\{\begin{matrix}\text{5 chances to have } d\\ \text{31 chances to have } \dfrac{4500d+27900x}{46656}\end{matrix}\right\}$ this which is worth $\frac{372780d+864900x}{1679616}$. This is therefore equal to x. And therefore x is equal to $\frac{10355}{22631}$. The part of player A is therefore $\frac{10355}{22631}$ of that which is put into the game. And the rest $\frac{12276}{22631}$ is the part of B. And the one is to the other as 10355 to 12276, which are the same numbers of Mister de Fermat.

In the second question where he supposes that player A plays first 2 times, and then player B 3 times, and thus in sequence player A 3 times: the method is completely similar, and I find also the same numbers as Mister de Fermat, but that it is necessary to transpose them. That is to say that the part of A is to the one of B, as 87451 to 72360 instead as he has put 72360 to 87451.

The third is, when three players A, B and C wager with all 52 cards that the one who will have sooner one heart will win, and that one supposes that A takes the 1st card, B the 2nd C the 3rd and thus consecutively until one has won.

There are 13 hearts among these 52 cards, this is why if it happened that all the other 39 were taken according to the said order without that a person had encountered a heart, then it would be the turn of A to take, and he would have won assuredly. When therefore C takes the 39th card, in the case where until then no person has encountered, it is certain that A will have 13 chances to have lost and 1 chance to have all that which is put into the game, which I will call d as before. Now to have $\left\{\begin{array}{l}\text{13 chances to have } 0\\ \text{and 1 chance to have } d\end{array}\right\}$ this is worth $\frac{1d}{14}$ by our theorem, from here I know that when B takes the 38th card, A will have $\left\{\begin{array}{l}\text{13 chances to have } 0\\ \text{and 2 chances to have } \dfrac{1}{14}d,\end{array}\right\}$ (This is when B fails to encounter a heart, because then it is to C to take the 39th.) which chances are worth $\frac{1}{105}d$.

When A takes the 37th, A will have therefore $\left\{\begin{array}{l}\text{13 chances to have } d\\ \text{and 3 chances to have } \dfrac{1d}{105}\end{array}\right\}$ this which is worth $\frac{1368}{1680}d$. Thus in referring back always to one card one will know in the end the part of A, when he takes the first of all.

And in the same manner will be found the part of B, and the rest will be that of C.

The 4th is, when two players play to prize with 40 cards and when the player A undertakes to restore prime, and B wagers that A will not succeed in the first four cards, one has said to me that to have prime it is to have 4 different cards, namely one of each sort. I find therefore that the part of A is to that of B as 1000 to 8139, so that one can well wager 8 against 1, that someone will not bring forth prime.

The 5th and last question[23] is, when two players play at piquet, and when the first undertakes to have 3 aces in his first twelve cards, and the other wagers that he will not have them. In order to resolve this, I will suppose, that he takes his 12 cards one by one, because it matters not at all. If it happens therefore that the one who undertakes having taken 11 cards, has already encountered 2 aces: there will be among the 25 cards which remain yet 2 aces. And hence he will have in this case 2 chances to have won, it is in order to have d and 23 chances to have 0, that is to say to lose. This which is worth $\frac{2}{25}d$.

When he has taken 10 cards, if he has encountered 2 aces, he will have therefore $\left\{\begin{array}{c}\text{2 chances to have } d,\\ \text{and 24 chances to have } \dfrac{2}{25}d\text{. That is in order to have only 2 aces in 11 cards.}\end{array}\right\}$ which chances are worth $\frac{49}{325}d$.

But when he has taken 10 cards if he has yet only 1 ace, He will have among the 26 remaining 3 aces, this is why he will have then

[23] See Letter No. 309.

$$\left\{\begin{array}{c}\text{3 chances to have } \frac{2}{25}d\text{, that is, in order to have 2 aces in 11 cards,}\\ \text{and 23 chances to have } 0,\ \text{that is, in order to have 1 ace in 11 cards,}\\ \text{because with this he would not know how to win}\end{array}\right\}\text{ which chances are}$$

worth $\frac{3}{325}d$.

When he has taken 9 cards, if he has 2 aces, he will have

$$\left\{\begin{array}{c}\text{2 chances to have } d,\\ \text{and 25 chances to have } \frac{49}{325}d,\ \text{that is,}\\ \text{in order to have only 2 aces in 10 cards}\end{array}\right\}\text{ which chances are worth } \tfrac{1875}{8775}d.$$

But having taken 9 cards if he has only 1 ace, he will have

$$\left\{\begin{array}{r}\text{3 chances to have } \frac{49}{325}d,\ \text{that is, 2 aces in 10 cards,}\\ \text{and 24 chances to have } \frac{3}{325}d\text{. that is, 1 ace in 10 cards,}\end{array}\right\}\text{ this}$$

which is worth $\frac{219}{8775}d$.

And at last if among his 9 cards he has yet no ace, he will have

$$\left\{\begin{array}{r}\text{4 chances to have } \frac{3}{325}d,\ \text{that is, 1 ace in 10 cards,}\\ \text{and 23 chances to have } 0\text{, that is, one ace in 10 cards, not one ace in 10 cards,}\\ \text{because then he would not know how to win.}\end{array}\right\}\text{ which}$$

chances are worth $\frac{12}{8775}d$.

Thus by this method to move back always by one card I will know in the end the part of player A, when he has taken no card, and that consequently he has not yet one ace. Which having taken off from d the rest will be the part of player B. This which it was necessary to find.

If I had been well informed of the state of the question in the game of chance that Mister de Fermat said to be the most difficult I would try also to resolve it. For those which I just treated, I pray you sir to do me the favor of communicating them to Mister Mylon And that I can know if that which Messrs. de Fermat and Pascal will have found of them will be conformed to that which I explicate of them. I desire also strongly to know if they do not serve themselves of the same theorem as me.. . .

SIR,

Your very humble and very obedient servant
CHR. HUYGENS OF ZUYLICHEM

No. 309[24]

Christiaan Huygens to P. de Carcavy
Appendix to No. 308

Proposed by Mister de Fermat

A and B play at piquet. A undertakes to have 3 aces in his first twelve cards. B wins if none, what is their share?

---

[24] Tome I. page 446-447.

There are 36 cards in piquet, of which 4 are some aces. 36 calculi quorum 32 nigri, 4 albi. caecus 12 capio, in quibus certo 3 albos esse.[25]

It is certain that A has the same chance, if one gives to him all his 12 cards at once, or one by one, and that it matters not whence one takes them.

In taking the 12th card (there are yet 25) if he has already 2 aces, he will have $\left\{\begin{array}{l}2 \text{ for } (1)\\ 23 \text{ for } 0\end{array}\right\}$ this is $\frac{2}{25}$. In taking the 11th card when he has 10 of them (when there are 26) if he has already 2 aces, he will have $\left\{\begin{array}{l}2 \text{ for } (1)\\ 24 \text{ in order to have } \dfrac{2}{25} \text{ that is 2 aces in 11 cards.}\end{array}\right\}$ that is $\frac{98}{650}$ let it be m. But if in taking the 11th when he has 10 cards, he has had yet only 1 ace he will have $\left\{\begin{array}{l}3 - \dfrac{2}{25}. \text{ This is 2 aces in 11 cards}\\ 23 - 0 \text{ this is 1 ace in 11 cards.}\end{array}\right\}$ this which let it be n.

In taking the 10th when he has 9 cards, (there are 27 of them) if he has had 2 aces, he will have $\left\{\begin{array}{l}2-(1)\\ 25-m\end{array}\right\} = p$.

If when he has 9 cards he has had 1 ace, he will have $\left\{\begin{array}{l}3-m\\ 24-n\end{array}\right\} = q$.

But having 9 cards if he has yet no ace at all, he will have $\left\{\begin{array}{l}4-n\\ 23-0\end{array}\right\} = r$.

Having 8 cards (remaining 28), if he has 2 aces, he will have $\left\{\begin{array}{l}2-(1)\\ 26-p\end{array}\right\} = s$. If he has had 1 ace, he will have $\left\{\begin{array}{l}3-p\\ 25-q\end{array}\right\} t$, if he has no ace at all, he will have $\left\{\begin{array}{l}4-q\\ 24-r\end{array}\right\} u$.

Having finally 2 cards (34 remaining) if he has 2 aces, he will have $\left\{\begin{array}{l}2-(1)\\ 32-x\\ \text{that is 2 aces in 3 cards}\end{array}\right\} = y$.

If he has had 1 ace, he will have $\left\{\begin{array}{l}3-x \text{ this is 2 aces in 3 cards}\\ 31-u \text{ this is 1 ace in 3 cards}\end{array}\right\} = \beta$.

If he has no ace at all, he will have $\left\{\begin{array}{l}4-f \text{ this is 1 ace in 3 cards}\\ 30-g \text{ this is 0 ace in 3 cards}\end{array}\right\} = \delta$.

Having therefore 1 card (35 remaining) if he has had 1 ace, he will have $\left\{\begin{array}{l}3-y\\ 32-\beta\end{array}\right\} \gamma$.

If he has no ace at all, he will have $\left\{\begin{array}{l}4-\beta\\ 31-\delta\end{array}\right\} \epsilon$.

Therefore previous to taking the first card, he has $\left\{\begin{array}{l}4-\gamma\\ 32-\epsilon\end{array}\right\} \zeta$.

quod querebatur.[26]

---

[25] 36 pebbles of which 32 black (unlucky), 4 white (lucky). Blindly I seize 12, in regard to which a certain 3 to be lucky.

[26] What was demanded.

No. 310[27]

Christiaan Huygens to Cl. Mylon
6 July 1656
Response to No. 306.

6 July 1656

To Mister MILON

SIR,

I have asked Mister de Carcavy to communicate to you that which I have written on the problems, which were in the extract which he had taken the trouble, to make me of the letter from Mr. de Fermat.[28] And you will see in what manner of analysis I serve myself in these sorts of questions which you desire to know. I assure you that the same afternoon[29] that I received your letter, I have found the solution of all, as for the method, not as for the calculation; which is so long in some of them that I have not wished to amuse myself pursuing it until the end. It would not be difficult to me to invent from it again one hundred others in this matter which would be much more difficult; but this would serve only to torment the mind and lose time, which is worth more to employ in the research of the things which it matters more to know. . . .

SIR,

Yours &c.

No. 319[30]

Christiaan Huygens to G.P. de Roberval
[27 July 1656][31]
Response to No. 311.

ROBERVAL

[Postscript:] It is a longtime that I have had no news at all of M.[32] I know not what I must think of this that neither he nor Mister de Carcavy respond to me at all since I have sent to them the solution of some questions on a matter of the games of chance, which Mister de Fermat had proposed to me. Among others he had this one, to determine what is the advantage of the one who playing at piquet wins if he will have 3 aces in his first 12 cards, which is rather difficult.

If you know of Carcavy I ask you from me to say nearly this &c.

[27] Tome I. page 448

[28] See Letter No. 301.

[29] *Translator's note*: The word is "apresdisnee."

[30] Tome I. page 464.

[31] The date is borrowed from an incomplete draft of this letter.

[32] Claude Mylon

No. 336[33]

P. de Carcavy to Christiaan Huygens
28 September 1656
Response to No. 308. Chr. Huygens responds in No. 342.

from Paris this 28th Sept. 1656

SIR,

It is already a long time that I have shown to Messrs. De Fermat and Pascal that which you have taken the trouble to send to Mister Mylon and to me touching on the parts, but I have not been able to give myself the honor to make response to you, the thing not having depended absolutely on me, and the convenience of these sirs not being always encountered with the desire that I had to satisfy you.

Mister Pascal serves himself of the same principle as you, and here is how he enunciates it.

If there are such number of chances as one will wish as for example ten, which give each three pistoles; and if there are two of them which give each 4 pistoles; and if there are three of them which each take off three pistoles; It is necessary to add all the sums together, and the chances together, and to divide the one by the other, the quotient is the required, that which returns to a like enunciation as yours.

But he does not see how this rule can be applied to the following example.

If one plays to six games, for example, of piquet, a certain sum and if one of the Players, has two, three, or four games, and if one wishes to quit the Game; what division it is necessary to make, when one has one game to point, or two, or three &c. to point, or else when one has two games and another one &c. And the said Gentleman Pascal has found the rule only when one of the Players has one game to point, or when he has two of them to point, (when one plays in many games,) but he has not the general rule, here is his enunciation.

It belongs to the one who has the first part of as much as one will wish for example of six out of the money of the loser, the product of as many of the first even numbers as one plays of games, except one, divided by the product of as many of the first odd numbers, the first product will be the stake of the loser, the second product will be the part of it which belongs to the winner. For example if one plays to 4 games, take the first three even numbers 2, 4, 6, multiply one by the other it is 48, take the first three odd numbers 1, 3, 5, the product is 15, which belongs to the winner out of the money of the loser if one has set each 48 pistoles, this rule serves for the first and the second game, the one who has two of them having the double of the one who has only one of them, he has the demonstration of it but which he believes very difficult.

Here is another proposition which he has made to Mister de Fermat which he judges without comparison more difficult than all the others.

Two players play with this condition that the chance of the first is 11, and that of the second 14, a third casts three dice for two of them, and when 11 arrives, the player makes a point and when 14 arrives, the second on his side marks one; the Players play to 12 points, but with the condition that if the one who casts the die brings back 11, and if thus the first marks a point if it happens that the die falls 14, the coup afterwards, the second marks not at all, but takes off one from the first, and thus reciprocally, so that if the die brings forth 11 six times, and the first has marked six points, if afterwards the die brings forth three times in sequence 14, the second will mark nothing but will take off three points from the first, if there arrives also afterwards that the die falls six times in sequence 14, there will

[33] Tome I. pages 492-494.

remain nothing to the first, and the second will have three points, and if he brings forth still eight times in sequence 14 without bringing forth 11, between the two, the second will have 11 points and the first nothing, and if he brings forth four times in sequence 11, the second will have only seven points, and the other nothing, and if he brings forth 5 times in sequence 14 he will have won.

The question seemed so difficult to Mister Pascal that he doubted if Mister de Fermat would come to the end of it, but he sent me immediately this solution. The one who has the chance of 11 against the one who has the chance 14, can wager 1156 against 1, but not 1157 against 1. And that thus the true ratio of this game was between the two, whence Mister Pascal having known that Mister Fermat had quite well resolved that which had been proposed to him, he gave me the true numbers in order to send them to him, and in order to witness to him that on his side he had not proposed to him a thing that he had not resolved before, here they are.

$$150094635296999121$$

$$129746337890625.$$

But that which you will find more important is that the said Gentleman de Fermat has the demonstration of it, as also Mister Pascal on his side, although there is appearance that they are themselves served of a different method. . . .

SIR,

Your very humble and obedient servant,
DE CARCAVY

No. 342[34]
Christiaan Huygens to P. de Carcavy
12 October 1656
Response to No. 336.

12 Oct. 1656

CARCAVY

SIR,

I am much obliged to you of the trouble that it has pleased you to take in sending to Mister Fermat the copy of my small work and the solution that I have given to the problems which he had proposed. I hope that Mister Pascal will have approved as well since you do not make me understand the contrary.

In the Theorem of which you say that this last serves himself likewise as myself there is some fault, which comes however not from him, but that I ascribe to the haste which you have had to describe it. Because assuredly Sir, you have wished to say, That having multiplied each number of chances, by the number of pistoles, that these chances give, it is necessary to add all the products together and the chances together and to divide one by the other; and that thus the quotient will be the required. And it is true that this the same enunciation as mine.

Now from this rule I serve myself also in the example which you have proposed, namely when one plays to 6 games or advantage, and that one of the two players has one game or two or 3 &c. to point. And although this is not with the same brevity that that of Mister Pascal brings to the case if one has one or two games to point; It is however quite universal, and I believe that in the other cases in which that of Mister Pascal extends not at all, it serves

[34]Tome I. pages 505-506.

itself of a same or of a similar, and that even without aid of such he is not arrived to his own which is in truth quite concise and good.

The proposition which he has made to Mister de Fermat appeared to me rather perplexing, but I have seen shortly that it was a question only of this, namely, that one of the players marking a point when there arrives 11 from three dice, and the other marking a point when there arrives 14, and the one of the two winning who first will have scored 12 points advantage over the other, it is necessary to determine the advantage of each of them. The problem being quite pretty in my opinion, and seeing that Mister Pascal had judged it so difficult that he doubted if Mister de Fermat could come to the end I have little to prevent myself from seeking also the solution of it, although you have sent me that which both have made of it. I myself am always served of the same theorem as above, and by the means of that there and of the algebra I have found the general rule for this question, which is quite simple as you will see. Being given such chances as we will wish of 2 or three or many dice, and any number that there be of the points which finish the game, it is necessary to see firstly how many chances there are for each of the chances or two other numbers in the same ratio. The numbers of these chances being multiplied each by itself as many times as there are points which finish the game, the products will have between them the required proportion of the advantages. For example in the case that Mister Pascal has proposed, there are 27 chances for the chance of 11, and 15 chances which give 14. Now as 27 to 15 thus is 9 to 5, it is necessary therefore to multiply the 9 and the 5 each 12 times by itself, because one plays to 12 points; the products are 282429536481 and 244140625, which I say expresses the true proportion of the advantages. Thus have they between them the same ratio as those of Mister Pascal who had 150094635296999121 and 129746337890625, and they are the smallest which it is possible to find. If the chance of the one is 10, and of the other 13, and if they play to 10 points, the advantages will be by this rule as 3486784401 to 282475249, and if the chances are 13 and 17, in playing to 12 points, the advantage of the one to that of the other will be precisely as 13841287201 to 1. This which will seem first rather strange.[35]

The method by which I have found the rule teaches me also at the same time to make the demonstration which will be however quite long.

Give me the indulgence I ask you to communicate all this to Mister Milon, and also to Mister de Roberval, since you see him sometimes. In order that I have no need to write many times a same thing. . . .

SIR,

Your very humble and obedient servant,
CHR. HUYGENS OF ZULICHEM

[35]Demonstration to Mylon [Chr. Huygens]..

No. 357[36]

Christiaan Huygens to Cl. Mylon

8 December 1656

Mylon responds in No. 366.

8 Dec. 1656

To Mister MILON

SIR,

. . .In sending lately to Mister de Carcavy[37] the rule which I have found for the resolution of a question in the matter of games of chance which Mister Pascal had proposed as the most difficult which he had encountered of this sort, I asked you to communicate it. But not having had response from him I fear that he will not have received my letter. You will oblige me Sir to make it known of me if it is thus (because without doubt you have seen more than once Mister de Carcavy since this time) so that by misfortune perhaps I am not accused of negligence without being guilty of it. . . .

No. 366[38]

Cl. Mylon to Christiaan Huygens

5 January 1657

Response to No. 357. Chr. Huygens responds in No. 370.

At Paris this 5th January 1657

SIR,

I was not in this city when Mister de Carcavy received your rule for the general Solution of the question of chance,[39] and when I returned from grape gathering he had gone to La Rocheguion from which he is to return only since three days. I have therefore been able to see him only yesterday morning. He asked me to give you his complements and his excuses. The quantity of affairs which occupy him do not permit him to dispose of his time as he would desire in order to make response to you. He says to me that your Method was admirable, that Mister Pascal had judged it as he, that he had not yet sent it to Mister Defermat because he was not then at Toulouse but at Chastres from which at present he believes him to return, that he would not fail to give it to the 1st postal courier. He has promised it to me when his country attire, where it is, will be unpacked. I can not await this time there in order to thank you for all your kindness, making me part of so much of good things. . . .

SIR,

Your very humble and very obedient servant, MYLON

[36] Tome I. pages 524-525.

[37] See Letter No. 342.

[38] Tome II. page 1.

[39] See the Letter No. 342.

No. 370[40]

Christiaan Huygens to Cl. Mylon

5 January 1657

Response to No. 366. Cl. Mylon responds in No. 371.

the 1 February 1657

To Mister MYLON

SIR,

I have been quite glad to be informed through this that it has pleased you to write to me on the 5th of January that Mister de Carcavy has received my long letter,[41] and that he and Mister Pascal have approved the rule that I have found. If one had not assured me when I was in Paris that this last had entirely abandoned the study of mathematics I would have tried by every means to make acquaintance with him.. . .

No. 371[42]

Cl. Mylon to Christiaan Huygens

2 March 1657

Response to No. 370.

At Paris this 2nd March 1657

SIR,

Although it was very difficult to approach Mister Pascal, and although he is completely retired in order to give himself entirely to devotion, he has not lost mathematics from view. When Mister de Carcavi can encounter him and when he proposes to him some question, he does not refuse him the solution of it and principally in the subject of the Games of chance which he is the first to bring forward. Not being so good as these two sirs, I have all the troubles in the world to see them, because their habits are in the Religions and in the affairs, and I visit these places there only very rarely. I have not been able to acquire the sentiment of Mister de Fermat touching your way of resolving the question of Chance, for me I find it very good and very simple. It returns to the composite ratio, because to multiply twelve times each of the terms 27, and 15. or 9. and 5. it is to have a dodecupled ratio of 27 to 15 and I find very reasonable since on the first Coup, the advantages of the Players (who draw one to 11 and the other to 14) are as 9 to 5, that one multiplies these advantages 12 times when one plays to 12 whole coups.. . .

---

[40] Tome II. page 7.

[41] This is the Letter No. 342.

[42] Tome II. pages 8-9.