APPENDIX I

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OEUVRES COMPLÈTE, TOME XIV. P. 92–95

If there remains 1 game to win to A and 1 to B, and 2 games to C, how much will the position be worth of each bettor if they have set each 2 écus into the game?

If C loses the first game he will lose 2 écus, if he wins it he will be clear, but it is two against one that he wins this first game, there are therefore two chances in order to lose 2 écus, and 1 in order to be clear. Hence I say that his part of 6 écus is $\frac{2}{3}$ écu.[1] Because taking these $\frac{2}{3}$ écu, he will find two others who will put $\frac{2}{3}$ écu against his $\frac{2}{3}$.[2] And [they] will play to who will have the total, namely 2 écus, in what will have same chance as before, namely 2 chances in order to lose his $\frac{2}{3}$ écus, that is to say 2 écus, adding that which he will have left to A and B: and one chance in order [93] to be clear, by winning from the two others each their $\frac{2}{3}$ écu. Because thus he will have 2 écus, as before as he had played against A and B. It follows from it that the positions of A and B are worth each $2\frac{2}{3}$ écus. And if we divide that which is in the game into 9 parts, A will take 4 of it. B 4. C 1.[3]

If there remains 1 game to win to A, 2 to B, and 2 to C, 6 écus in the game.

If A wins the first he wins 4 écus. If B or C win it, A wins $\frac{2}{3}$ écus by the preceding, therefore A has 2 chances in order to win $\frac{2}{3}$ écus and 1 chance in order to win 4 écus. I say that of the 6 écus his part is $3\frac{7}{9}$ écus. That is to say that he wins $1\frac{7}{9}$ écus, because taking beyond his 2 écus which he had set, still $1\frac{7}{9}$ écus that I say he wins he will set $\frac{10}{9}$ écus against two others who put $\frac{10}{9}$ écus each in it, in order to play who will win all. And thence he will have 1 chance in order to win $\frac{30}{9}$ écus which with the $\frac{6}{9}$ écus which he will have set aside, will make him win 4 écus and 2 chances in order to win only $\frac{6}{9}$ it is $\frac{2}{3}$ écus which he will have set in part. Therefore B and C will have of the 6 écus each $1\frac{1}{9}$ écus. And if one divides that which is in the game into 27 parts, A will take 17, B and C each 5.[4]

If there remains 1 to A, 2 to B, 3 games to C, 6 écus in the game. How much is the position of each worth.

Translated by Richard J. Pulskamp, Department of Mathematics and Computer Science, Xavier University, Cincinnati, Ohio. Printed on February 1, 2009.

[1]See Prop. III. Huygens, instead of corresponding to this Proposition, follows a demonstration which is independent.

[2]In this manner Huygens makes the demonstration of his solution rest on the axiom which he has announced at the beginning of his Treatise.

[3]Compare the solution of Prop. VIII.

[4]Compare the two cases to the Table for three players.

If A wins the first game he wins 4 écus. If he loses it he has one chance in order to win $\frac{2}{3}$ écus[5] and one in order to win $1\frac{7}{9}$ écus[6], which are worth as much as he was [94] assured to win (in case of loss of the said first game) $\frac{11}{9}$ écus, which is worth the half of the said $\frac{2}{3} + 1\frac{7}{9}$. Now he has two chances to lose the first game, and one chance in order to win it, therefore he has one chance in order to win 4 écus and 2 in order to win $\frac{11}{9}$ écus. I say that he wins $2\frac{4}{27}$ or else $\frac{58}{27}$ écus. Because taking this and setting aside $\frac{11}{9}$ écus he will set the remaining which is $\frac{25}{27}$ against two others who will put each as much. And thus he will have 1 chance in order to win $\frac{25}{9}$ écus that is to say (adding $\frac{11}{9}$ which he is reserved himself) $\frac{36}{9}$ or 4 écus and 2 chances in order to win only the $\frac{11}{9}$ écus which he is reserved himself. His part therefore of the 6 écus is $2 + 2 + \frac{4}{27}$ écus it is $4\frac{4}{27}$ écus.[7]

In order to know how much will be B. I say, if B wins the first game he will win $\frac{2}{3}$ écus[8] *per primam*. If he loses it, he courts equal fortune to lose 2 écus or to lose $\frac{8}{9}$ écus *per secundam*, which is as much as if losing this first game he would lose $\frac{13}{9}$ écus namely the half of $2 + \frac{8}{9}$. Now there are 2 chances in order to lose the 1st game and 1 chance in order to win it. Therefore there are 2 chances in order to lose $\frac{13}{9}$ écus and 1 chance in order to win $\frac{2}{3}$ écus. I say that he will have of the 6 écus $\frac{34}{27}$ or $1\frac{7}{27}$ écus.[9] Because taking $\frac{34}{27}$ écus he will set $\frac{19}{27}$ écus in it against two others who each will put in it as much and thus will have 1 chance in order to win $\frac{19}{9}$ écus, which with $\frac{15}{27}$ écus or $\frac{5}{9}$ [95] that he has reserved aside are $\frac{24}{9}$ or $2\frac{2}{3}$, that is to say that he will win $\frac{2}{3}$ écus. And two chances in order to have only the $\frac{5}{9}$ écus you reserve, it is to lose $\frac{13}{9}$ écus of it because he had put 2 écus into the game.

It follows from it that C will have of the $6\frac{16}{27}$ écus.[10] Therefore if one divides the whole into 81 parts, A will take 56, B 27, C 8.[11]

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[5]Evidently the question here is of the case where it is B who wins. Then there remains to A 1 game, to B 1 and to C 3 games to win. Huygens should have therefore calculated the "chances" of this case and he would have found $\frac{8}{9}$ instead of $\frac{2}{3}$ écu. It is only inadvertantly that he has taken this last number which agrees in the case, treated at the end of this piece, where there lacks to A 1 game, to B 1 and to C 2 games.

[6]One reads in the margin "by the preceding;" see, indeed, the solution of the case where there remains to A 1 to B 2 and to C 2 games to win.

[7]In truth $4\frac{2}{9}$.

[8]Same confusion between the cases 1, 1, 2 and 1, 1, 3. One must replace, as before, $\frac{2}{3}$ by $\frac{8}{9}$.

[9]In truth $1\frac{1}{3}$.

[10]In truth $\frac{4}{9}$.

[11]In the "Table for three players" this erroneous solution is replaced by the correct according to which of 27 parts A will take 19, B 6 and C 2.