# Solution to Huygen's First Exercise[1]

Fermat posed this question to Huygens in the letter of June 1656, No. 301. Huygens gives the solution in the 6 July 1656 letter to Carcavi, No. 309. Player A wins if he casts a 6 with two dice before Player B casts a 7. The turns of the Players proceed as ABBAABBAA...

**Solution:** Player A casts a 6 with probability $p = 5/36$. Player B casts a 7 with probability $r = 6/36$. Put $q = 1 - p$ and $s = 1 - r$. Let S denote a successful or winning cast and F a failure. Player A wins for the following sequences S, FFFS, FFFFS, FFFFFFFS, FFFFFFFFS, ... which occur with probabilities $p$, $qs^2p$, $q^2s^2p$, $q^3s^4p$, $q^4s^4p$,... respectively. It is apparent that there are two subsequences (one where the exponents of $q$ and $s$ differ by 1, and the other where their exponents are equal) which may be summed easily simultaneously. Thus the probability that A wins is given by

$$p + p(qs^2 + q^2s^2) \times \sum_{k=0}^{\infty} (qs)^{2k} = \frac{10355}{22631}.$$

The probability that B wins is thus $\frac{12276}{22631}$. The advantage of A to B is 10355 to 12276.

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