Appendix III[1]

Chr. Huygens

Pages 102-107 T. XIV

[1665]

J[an] has 2 white tokens and 1 black, but P. 1 white and 2 black. And each in his turn chooses blindly one of his tokens. The one who obtains a black token must add a δ (ducat) to the stake, but the one who obtains a white token receives all that which has been set. And J. chooses first, when there is yet nothing in the stake. One demands how much is the advantage or the disadvantage of J. at the beginning of he game. Resp. J wins $\frac{207}{343}$ of a ducat.

$+a$	is the advantage of	P.	when he has set	0	against	0	and when	J.	must choose.[2]
$-b$		Jan		1		0		P.	
$-c$		P.		1		1		J.	
$-d$		Jan		2		1		P.	
$-e$		P.		2		2		J.	
$-f$		Jan		3		2		P.	
$-g$		P.		3		3		J.	
$-h$		J.		4		3		P.	...[3]

[1] This appendix is taken from pages 45-47 of Manuscript C; these pages were numbered 1-3 by Huygens. One finds there the solution of a problem posed by Huygens in a letter to Hudde of 10 May 1665. We add that this problem was a modification of another problem proposed by Hudde to Huygens in the letter of 5 May 1665.

[2] We remark that Huygens has therefore supposed, in beginning these calculations, that the advantage would be found on the side of P. , but that the calculations have taught the contrary to him.

[3] Further down in the manuscript these previous definitions are continued further by introducing the letters i, $\aleph$, $\beth$, $\daleth$ but these latter definitions have been suppressed.

[103]

$+k$	is the advantage of	J.	when he has set	0	against	0	and when	P. must choose.
$-l$		P.		1		0		J.
$-m$		J.		1		1		P.
$-n$		P.		2		1		J.
$-o$		J.		2		2		P.
$-p$		P.		3		2		J. $\ldots$[4]

1. $-b$ and 2. $+k$[5]

$-a = \frac{-b+2k}{3}$

$-3a + b = 2k$

$a = \frac{b-2k}{3}$

$-b = 1. -\delta \quad 2. +c$[6]

$-b = \frac{-\delta+2c}{3}$

$\frac{1}{3}b = \frac{-2c+d}{9}$

$-c = 2. -\delta \quad 1. +d$

$\frac{2}{9}c = -\frac{2}{27}d + \frac{4}{27}\delta$[7]

$-d = 1. -2\delta \quad 2. +c$

$\frac{2}{27}d = \frac{4}{81}\delta - \frac{4}{81}e$

$-e = 2. -2\delta \quad 1. +f$

$-\frac{4}{81}e = -\frac{16}{243}\delta + \frac{4}{243}f$

$-f = 1. -3\delta \quad 2. +g$

$\frac{4}{243}f = \frac{12}{729}\delta - \frac{8}{729}g$

$-g = 2. -3\delta \quad 1. +h$

$-\frac{8}{729}g = -\frac{48}{2187}\delta + \frac{8}{2187}h$

$-h = 1. -4\delta \quad 2. +i$

$\frac{8}{2187}h = \frac{32}{6561}\delta - \frac{16}{6561}i$

[4] These definitions are also continued further down by introducing the letters q, r, t and v

[5] This notation ($1. -b$ and $2. +k$) indicates, as Huygens will explain expressly in another Part which we reproduce further down, (page 124) that J[ean] has (at the beginning of the game) one chance to obtain $-b$ and two to obtain $+k$. One sees therefore that Huygens supposes that the game will continue if Jean takes a white token on the first coup. Now, this assumption has given place to a new misunderstanding between Hudde and him, since Hudde considered that the game was ended in this case. According to this last interpretation one would have therefore $k = 0$.

[6] Indeed, it is evident that the advantage of one of the players is always equal to the disadvantage of the other.

[7] Suppressed, here and in the following, are some calculations entirely analogous to those which precede.

[104]

$-i = 2. - 4\delta \quad 1. + \aleph$
$-\frac{16}{6561} i = -\frac{128}{[19683]}\delta + \frac{16}{[19683]}\aleph$

$\aleph = 1. - 5\delta \quad 2. + ℸ$
$\aleph = \frac{-5\delta + 2ℸ}{3}$

$k = 2. + l \quad 1. - a$
$\frac{2}{3}k = \frac{4}{9}l - \frac{2}{9}a$

$-l = 2. - \delta \quad 1. + m$
$\frac{4}{9}l = \frac{8}{27}\delta - \frac{4}{27}m$

$-m = 1. - \delta \quad 2. + n$
$\frac{4}{27}m = \frac{4}{81}\delta - \frac{8}{81}n$

$-n = 2. - 2\delta \quad 1. + o$
$-\frac{8}{81}n = -\frac{32}{243}\delta + \frac{8}{243}o$

$-o = 1. - 2\delta \quad 2. + p$
$\frac{8}{243}o = \frac{16}{729}\delta - \frac{16}{729}p$

$-p = 2. - 3\delta \quad 1. + q$
$-\frac{16}{729}p = -\frac{96}{2187}\delta + \frac{16}{2187}q$

$-q = 1. - 3\delta \quad 2. + r$
$\frac{16}{2187}q = \frac{48}{6561}\delta - \frac{32}{6561}r$

$-r = 2. - 4\delta \quad 1. + s$
$-\frac{32}{[6561]}r = -\frac{256}{[19683]}\delta + \frac{32}{[19683]}s$

$-s = 1. - 4\delta \quad 2. + t$
$\frac{32}{[19683]}s = \frac{128}{[59049]}\delta + \frac{64}{[59049]}t$

$-t = 2. - 5\delta \quad 1. + v$
$-t = \frac{-10\delta + v}{3}$

$$\begin{array}{ccc} & \text{A} & \text{B} \\ a = & \dfrac{1}{3}b & -\dfrac{2}{3}k \end{array}$$ [8]

[8] See the first equation obtained on page 103. In that which follows Huygens will designate by A the first term $\frac{1}{3}b$, by B the second term $-\frac{2}{3}k$ of the second member of this equation.

[105]

A

$$\frac{1}{3}b = \frac{1}{9}\delta - \frac{2}{9}c$$

$$-\frac{4}{27}\delta + \frac{2}{27}d$$ [9]

$$+\frac{4}{81}\delta - \frac{4}{81}e$$

$$-\frac{16}{243}\delta + \frac{4}{243}f$$

$$+\frac{12}{729}\delta - \frac{8}{729}g$$

$$-\frac{48}{2187}\delta + \frac{8}{2187}h$$

$$\left[-\frac{16}{6561}i\right] \qquad +\frac{32}{6561}\delta - \frac{16}{6561}i$$

$$-\frac{128}{[19683]}\delta + \frac{16}{[19683]}\aleph$$

$$+\frac{80}{[59049]}\delta - \frac{32}{[59049]}\daleth$$ [10]

[9] Huygens wishes to indicate thus that one can replace $-\frac{2}{9}c$ by $-\frac{4}{27}\delta + \frac{2}{27}d$; see the calculations which one finds on pages 103-104.

[10] As we will see, the solution of Huygens rests on the assumption that this last term (and also the one of the expression for B) approaches indefinitely to zero. This admitted, the question is no more but to sum the infinite series formed by the terms which contain δ.

B

$$-\frac{2}{3}k = \frac{2}{9}a - \frac{4}{9}l$$
$$-\frac{8}{27}\delta + \frac{4}{27}m$$
$$+\frac{4}{81}\delta - \frac{8}{81}n$$
$$-\frac{32}{243}\delta + \frac{8}{243}o$$
$$+\frac{16}{729}\delta - \frac{16}{729}p$$
$$-\frac{96}{2187}\delta + \frac{16}{2187}q$$
$$\left[-\frac{32}{6561}r\right] \qquad +\frac{48}{6561}\delta - \frac{32}{6561}r$$
$$-\frac{256}{[19683]}\delta + \frac{32}{[19683]}s$$
$$+\frac{128}{[59049]}\delta - \frac{64}{[59049]}t$$
$$-\frac{640}{[177147]}\delta + \frac{64}{[177147]}v$$

[106]

THEOREM.

If descending magnitudes are in continuous geometric ratio the greatest will be with the all remaining to infinity to the greatest alone as the greatest is to the excess of the maximum beyond the following. Therefore if they are as 4 to 1 the greatest will be with all to the greatest as 4 to 3.

If they are as 9 to 2, the greatest will be with the whole to the greatest as 9 to 7, or the greatest with the whole will be $\frac{9}{7}$ of the greatest.

$$\left.\begin{array}{cccc} \frac{4}{81} & \frac{8}{729} & \frac{16}{6561} & \frac{32}{59049} \\ & \frac{8}{729} & \frac{16}{6561} & \frac{32}{59049} \\ & & \frac{16}{6561} & \frac{32}{59049} \\ & & & \frac{32}{59049} \end{array}\right\}\text{the } + \text{ of B}^{11}$$

This[12] sequence is $\dfrac{2}{9}$ of the preceding

$\dfrac{9}{7}$ [of] $\dfrac{4}{81}$ [=] $\dfrac{4}{63} =$ the first series

$\dfrac{9}{7}$ of $\dfrac{4}{63}$ [=] $\dfrac{4}{49}$ = [the sum] of all the series

that is to say = the positive terms of B.

the $+$ of B to the $-$ of B as 1 to 6
the $-$ of B to the $-$ of A as 2 to 1
the $-$ of A to the $+$ of A as 4 to 3

[11] Indeed, the sum of all these numbers is equal to that of the first four positive coefficients of δ in the series B which results from the development of $-\frac{2}{3}k$.

[12] That is to say in the series which one finds to the side.

[107]

$$\begin{array}{lll}
\text{the } + \text{ of B are} & = +\dfrac{4}{49}\delta & \\
\text{therefore the } - \text{ of B} & = -\dfrac{24}{49}\delta & \\
\text{therefore the } - \text{ of A} & = -\dfrac{12}{49}\delta & \\
\text{therefore the } + \text{ of A} & = +\dfrac{9}{49}\delta & \\
 & -\dfrac{23}{49}\delta & \text{to which adding } \dfrac{2}{9}a \text{ which} \\
 & \dfrac{2}{9}a & \text{are contained under B} \\
\hline
 & \dfrac{2}{9}a - \dfrac{23}{49}\delta = a; & \dfrac{207}{343}\delta = -a
\end{array}$$ [13]

[13] This is the result enunciated at the beginning of this Piece. See page 102.