ABRÉGÉ DES COMBINAISONS

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Hazards

It is the custom in Genoa to elect, or rather to draw by lot each year from among one hundred Senators five persons who must have the principal Offices of the Republic.

This has given rise to some wagers which are made each year concerning those on whom the lot will happen. There are found some Bankers who will promise up to twenty thousand pistoles for one that one will give them if the lot falls on the 5 who one will have named; five or six thousand, if there are only 4 of the 5 that one will have named of them; & 5 or 6 hundred, if there are 3. Ordinarily they give nothing for one or for two. One asks what are the hazards for the Banker & for the Bettor, & what profit the Banker can have on this business.

It is necessary firstly to settle that which the Banker must give, if the lot falls on those who one will have named, or on some of them.

We suppose that the Banker gives 20000 pistoles for one, if the lot falls on the 5 who one will have named; 5000 if there are only 4 of them; 300 if there are 3; & 4 if there are only 2 of them.

One will see firstly in how many ways the 5 who one must draw in the lot can come.

It is necessary to multiply 100 by 99, the product by 98, by 97 & by 96; but because the product of these 5 numbers contains also the order in which these five persons are drawn, it is necessary to divide by 120, which is the combination of five things; or else to multiply only 80 by 97, 98 & 99, which is the same thing as to multiply the five numbers by one another, & to divide the product by 120. One will have 75287520, which are all the ways in which five Tickets can be drawn or taken in 100.

It is necessary now to see what are the hazards of the Bettor.

The five that one has named can happen only in one way: he will have therefore only one hazard for him, & 75287519 for the Banker; & because the Banker gives 20000 for 1, it is necessary to multiply 1 by 20000, therefore for 20000 of hazard that the Bettor has, the Banker has 75287519 of them, which being divided by 20000 gives $3764\frac{3}{4}$, the proportion of the hazards is therefore as 1 to $3764\frac{3}{4}$.

In order to have the hazards of 4, it is necessary to take five times 95, which are 475, which it is necessary to subtract from all the hazards, or rather from those which the Banker has on the chances of 5. One will subtract therefore 475 from 75287519, there will remain 75287044 for the hazards of the Banker: but because he gives 5000 for one, it is necessary to divide 75287044 by 5000, & one will have a little more than $15057\frac{2}{5}$ for the hazards of the Banker, & 475 for those of the Bettor; & dividing the one by the other, there will come a little less than $31\frac{7}{10}$, for the hazards of the Banker, & one for those of the Bettor.

In order to have the hazards of three persons in the five that one has named, one will multiply by 10 the triangle of 94, which is 4465; one will have therefore 44650 for the hazards of the Bettor, which being subtracted from the hazards that the Banker has had for four, namely of 75287044, there will remain 75242394 for the hazards of the Banker, which it is necessary to divide by 300, because he gives 300 for 1, & one will have a little less than 250808 for the hazards of the Banker, which being divided by 44650, one will have the proportion of hazards of the Banker, & of the Bettor, as a little more than $5\frac{3}{5}$ to 1.

There remains to see the hazards for 2. In order to have those of the Bettor, one will multiply the tetrahedron of 93, namely 138415 by 10, & one will have 1384150, that it is necessary to subtract from the hazards that the Banker has had on 3, namely from 75242394, there will remain 73858244, which it is necessary to divide by 4, because the Banker gives 4 for 1, & one will have 18464561 for the hazards of the Banker; & dividing them by the hazards of the Bettor, which are 1384150, one will find that the hazards of the Banker & of the Bettor are between them as a little less than $13\frac{1}{2}$ to 1.[1]

One can also consider the hazards of 1, that is, if there comes some one of the five who one has named. It will be necessary to multiply by 5 the triangle-triangle or fourth power triangular of 92, which is 3183545, the product is 15917725, which it is necessary to subtract from the hazards of the Banker for 2, namely from 73858244, there will remain 57940519, which are the hazards of the Banker: but because he gives nothing, when there comes only one of the five that one has named, one will divide 57940519 by 15917725, & the Banker will have yet $3\frac{16}{25}$ of hazard on 1 that the Bettor will have; but this hazard is only to the profit of the Banker, & the Bettor has nothing.

Having all these hazards, it is necessary to assemble them. And firstly if the lot falls on the five who have been named, the Bettor has 20000. If there come four of them, there are 475 hazards for the Bettor, which being multiplied by 5000 that the Banker must give, if there arrives some one of 475 hazards, there will be 2375000 hazards for the Bettor.

If the lot falls on three of those who have been named, the hazards of three are 44650, which you multiply by 300, that the Banker must give for each of these hazards, this will be 13395000 hazards for the Bettor, if there come three of the five who he has named.

If the lot falls only on two, one has found that the hazards of two are 1384150, which multiplied by four give 5536600 hazards for the Bettor. All these hazards together amount to 21326600; & these are the hazards of the Bettor.

In order to have the hazards of the Banker, it is necessary to assemble all the hazards of the Bettor, which are 1, 475, 44650 & 1384150, the sum is 1429276, which subtracted from all the hazards which are in all 75287520, there will remain 73858244 for the hazards of the Banker, the hazards of the Bettor will therefore be to those of the Banker as 21326600 to 73858244, or in least terms, as 183850 to 636709, which is as 1 to a little less than $3\frac{1}{2}$, or correctly as 1 to $3\frac{85159}{183850}$.

Hazards	
20000	of 5
2375000	of 4
13395000	of 3
5536600	of 2
21326600	Sum of the hazards of the Bettor.

Hazards	
1384150	of 2
44650	of 3
475	of 4
1	of 5
1429277	Sum of the hazards of the Banker.

[1]*Translators note*: This ratio is 13.34 to 1.

This would be then the hazards of the Banker & the Bettor, if the Banker received nothing from those to whom the lot is favorable: but because beyond the advantage which he has in the hazards, he has again a pistole from each of those to whom the hazards are able to happen, there is for him all the hazards of five persons chosen from 100, namely 75287520, the proportion of the hazards of the Bettor is therefore to those of the Banker, as 21326600 to 75287520, or as 533165 to 1882188, that is as 1 to a little more than $3\frac{1}{2}$.

But because ordinarily one gives nothing for 2, it is necessary to subtract the hazards of 2, which amount to 5536600, from the hazards of the Bettor, the remainder will be 15790000, which are to the hazards of the Banker, as 394750 to 1882188, or as 1 to a little more than $4\frac{3}{4}$.

Here are the foundations & the reasons for this operation.

Firstly, in order to know in how many ways one can choose five things in 100, one multiplies by one another the five numbers 100, 99, 98, 97 & 96, & one divides the last product by the order of five things.

If one takes only one thing in 100, it is certain that one can make it only in 100 ways. But if one takes two of them, since the first is taken in 100 ways, after each of the 100 one can set what one will wish of the 99 remaining; but one sees here that the order is included, because each of the 100 will be in each the first & the last choice, it will be necessary therefore to divide by two, namely by the order of two things, the product of 99 by 100.

If one chooses three things in 100, because two things are taken in 9900 ways, which is the product of 100 by 99, & since there remain 98 of them, one can choose any of those 98 who remain, & join it to each of 9900 ways of which one has chosen two things: the product of 9900 by 98 will contain the diverse ways to choose three things in 100, the order contained.

For the same reason, in order to choose four things, it is necessary to multiply the last product, which is 970200, by 97 which remain, & for five things to multiply again this last product, which is 94109400, by 96, & to divide the product 9034502400 by 120, which is the order of five things, because by this construction each 100 things hold alternately each of the five ranks which are in five things, namely the first, the second, third, fourth & last.

For the hazards of the Bettor, one multiplies 95 by 5, in order to know in how many ways there can come four of the five who he has named; for since he lacks one of them, he can lack any of the five, & in its place there can come one of the 95, of whom he has not named any: it is necessary therefore to multiply 95 by 5. It is true that the four being subtracted from 100, there remain 96: one does not multiply however by 96, because the fifth of the named being found in the number of hazards of the Bettor, it would count twice to the Bettor.

If three of the five come who have been named, the hazards are found by multiplying by 10 the triangle of 94.

One multiplies by 10, because three things can be chosen from five in ten ways, as it has been explained above, when one has seen in how many ways one can choose five things in 100, because one will multiply 5, 4, 3, by one another, & one will divide the product 60 by the order of 3, which is 6.

One multiplies by the triangle of 94, because in the five which one draws at random, there being only three of the five who the Bettor has named, the two others must be of the 95 others. Now from all these things or hazards, the first of these 95 will be found with each of the 94 others. After the second of these same 95 will be found with each of the 93 others. The third with each of the 92; & so forth to 94 which will be found with the last of

the 95: now these numbers together are the triangle of 94, because it would be necessary to join 94 with 93, 92, 91, &c. to 1.

By the same reasoning one will see why in order to have the hazards of two, one multiplies by 10 the tetrahedron of 93.