SOLUTION D'UN PROBLÈME SUR LES JEUX DE HASARD

ALEXIS FONTAINE DE BERTINS
MÉMOIRES DONNÉ À L'ACADÉMIE ROYALE DES SCIENCES NON IMPRIMÉS DANS LEUR TEMPS.
(1764)

Pierre proposes to me to play with him at *heads* or *tails*, & he offers to give to me an écu, if I bring forth heads at the first coup; two, if I bring it forth only at the second; four, if I bring it forth only at the third; eight, if I bring it forth only at the fourth; sixteen, if I bring it forth only at the fifth, & thus in sequence by doubling; to find the sum that I must give to him at the beginning of each game in order that the game be equal.

In following the ordinary rules of the games of chance, one found that I must give to Pierre an infinite sum, & it was necessary to resolve this difficulty.

I suppose that we play, Pierre & I, silver on the game, so that, anything which arrives, I am never able to wager beyond of that which is in the game; it is evident that this condition changes nothing in the enunciation of the Problem, because one can suppose that Pierre puts all his wealth into the game.

Let the sum of the écus that Pierre puts into the game be $= x$, that which I must put, in order that the game be perfectly equal, $= y$, the silver of the game will be $= x + y$, & one will have $y = \frac{(n+1)2^n + x}{2^{n+1} - 1}$.

As for the positive whole number n, one will determine by the condition that x must be $> \frac{2^{n+1} - n - 2}{2}$ & $< \frac{2^{n+2} - n - 3}{2}$.

I suppose, for example, that Pierre puts a million into the game, that is to say, that $x = 333333 + \frac{1}{3}$, one will have $333333 + \frac{1}{3} > \frac{2^{n+1} - n - 2}{2}$ & $< \frac{2^{n+2} - n - 3}{2}$; therefore $n = 18$; therefore $y = \frac{19.2^{18} + \frac{1000000}{3}}{2^{19} - 1} = 30^{\mathrm{l}}\ 8^{\mathrm{s}}\ 2^{\mathrm{d}}$ nearly.[1]

If it is my stake which is given, & if one wishes to have that of Pierre, one will have $x = 2^{n+1}y - (n+1)2^n - y$, & n will be $> 2y - 3$ & $< 2y - 2$.

If y is a whole number, you will make $n = 2y - 3$, or $n = 2y - 2$, & you will have $x = 2^{2y-2} - y$.

In order to play at this game, it would be only to have one die with six faces, of which three would be black, & the three others white; one would have moreover a large & heavy coin with two faces, one black & the other white; one would overturn this coin at each time before casting the die, & one would agree that as often as the color of the die & that of the coin will be the same, the game will be finished. I suppose that in beginning the game, the superior face of the coin is black.

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[1] *Translator's note*: In French currency at this time we have 3 livre = 1 écu, 20 sou = 1 livre, and 12 douzaine = 1 sou. Fontaine converts $x = 1{,}000{,}000$ livres into $333,333\frac{1}{3}$ écus and computes $y = 10.1358\ldots$ écus. Multiplying by 3, we have $y \approx 30.4074$ livres. Multiplying the fractional part of this by 20 gives approximately 8.148 sou. Multiplying the fractional part of this figure by 12 yields approximately 1.8 douzaine. Hence we have 30 livre, 8 sou and nearly 2 douzaine.

I cast the die; if I bring forth black, Pierre will give me an écu. If I bring forth white, one will make a mark in order to remember that I have played a coup; one will overturn the coin, & its color will be white.

I cast the die; if I bring forth white, Pierre will give me two écus. If I bring forth black, one will mark that I have played two coups; one will overturn the coin, & its color will be black.

I cast the die; if I bring forth black, Pierre will give me four écus. If I bring forth white, one will mark that I have played three coups; one will overturn the coin, & its color will be white.

I cast the die; if I bring forth white, Pierre will give me eight écus. If I bring forth black, one will mark that I have played four coups; one will overturn the coin, & its color will be black.

I cast the die; if I bring forth black, Pierre will give me sixteen écus. If I bring forth white, one will mark that I have played five coups; one will overturn the coin, & its color will be white, &c.

In order to not have to calculate at the beginning of each game, it will be necessary that Pierre always keep the same sum in the game.

Let, for example, $y = 10$ écus, one will have $x = 2^{2\cdot 10-2} - 10 = 2^{18} - 10 = 262134$ écus; thus, if Pierre keeps in the game 262134 écus, at the beginning of each game, I will put 10 écus.

Let $y = 11$ écus, one will have $x = 2^{2\cdot 11-2} - 11 = 2^{20} - 11 = 1048565$ écus. If Pierre keeps in the game 1048565 écus, I will put 11 écus at the beginning of each game.