ENCYCLOPÉDIE METHODIQUE

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CARTES

CARDS, *Problem on the cards*. (*Arithmetic*.) Pierre holds eight cards in his hands which are: an ace, a deuce, a three, a four, a five, a six, a seven & an eight, which he has shuffled: Paul wagers that dealing them one after the other, he will guess them in step as he will deal them.[1]

One demands how much must Pierre wager against one that Paul will not succeed in his enterprise?

By the wording of the question, one supposes that Paul wagers to deal all the *cards* one after the other, without returning them to the deck after having drawn them, & without missing a single time at guessing the card correctly he will draw.

In this case, by following the ordinary rules of probabilities, the expectation of Paul at the first draw is $\frac{1}{8}$, at the second $\frac{1}{7}$, whence it follows that his expectation for the first two draws is $\frac{1}{8} \times \frac{1}{7}$; & in fact, it is easy to see that the first draw having eight possible cases, & the second seven, the combination of the two will be 8×7 draws, of which there is only one which makes Pierre win, the one where he will guess correctly two times in sequence. For the same reason, the expectation of Paul for three draws will be $\frac{1}{8} \times \frac{1}{7} \times \frac{1}{6}$; for four, $\frac{1}{8} \times \frac{1}{7} \times \frac{1}{6} \times \frac{1}{5}$; & for seven (because he is not able to have eight of them, expecting that after seven draws there remain no more *cards* to draw, & there is no more deck), it will be $\frac{1}{8} \times \frac{1}{7} \cdots \times \frac{1}{2}$; therefore the wager of Pierre will be to that of Paul as $8 \times 7 \times \cdots 2 - 1$ is to 1, that is to say, as $56 \times 720 - 1$ is to 1; or as 40319 is to 1.

If Paul wagered to produce or to guess correctly on one of the seven draws alone, his expectation would be $\frac{1}{8} + \frac{1}{7} \cdots + \frac{1}{2}$, & consequently the wager of Pierre to that of Paul, as $\frac{1}{8} + \frac{1}{7} \cdots + \frac{1}{2}$ to $1 - \frac{1}{8} - \frac{1}{7} \cdots - \frac{1}{2}$.

If Paul wagered to produce correctly in the first two draws alone, his expectation would be $\frac{1}{8} + \frac{1}{7}$, & the ratio of the wagers that of $\frac{1}{8} + \frac{1}{7}$ to $1 - \frac{1}{8} - \frac{1}{7}$.

If he wagered to produce correctly in any two draws, his expectation would be $\frac{1}{8} \times \frac{1}{7} + \frac{1}{8} \times \frac{1}{6} \cdots + \frac{1}{8} \times \frac{1}{2} + \frac{1}{7} \times \frac{1}{6} \cdots + \frac{1}{7} \times \frac{1}{2} \cdots + \frac{1}{6} \times \frac{1}{5}$ &c.

Another Problem

One demands how much are the odds against 1 that drawing five *cards* in a game of piquet, composed of thirty-two, one will not draw one major indeterminate fifth, without naming the color, either hearts, or diamonds, or spades or clubs?[2]

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[1]*Translator's note*: Thus is introduced the problem of derangements.

[2]*Translator's note*: The deck consists of the cards 7 through Ace, inclusive. A fifth is a sequence of cards of the same suit. A major fifth is the sequence of highest ranking cards, namely Ace, King, Queen, Jack and 10.

In order to solve the proposed question, it is necessary first to find in how many ways thirty-two *cards* are able to be taken five by five, & one will find by the common rules of combinations, that this number of times is the product of the five numbers 28, 29, 30, 31, 32; this product being divided by the product of the five other numbers $1, 2, 3, 4, 5$; or by 120: that is to say, that the number of times sought is the product of the numbers 28, 29, 31, 8, or 201376. Now, as there are four major fifths, it is necessary to remove from this number 4 of 201376, this which will give 201372, & the odds will be 4 against 201372, or 1 against 50343 that one will not draw a major fifth at will.

If the question is of any one fifth, as there are in all sixteen fifths, namely four of each color, the wager must be 16 against 201376 less 16, or of 16 against 201360, or of 1 against 12585. (*M. d'Alembert*)

Translator's note: Here follows a history of the card deck drawn from the work of Fr. Mènetrier. It is not due to d'Alembert.